Optimal Disclosure Windows*

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Abstract

I study a dynamic disclosure game where an agent controls a window of time over which information flows to the decision maker, but does not control the content of the information disclosed. In equilibrium, the agent may have an incentive to delay the start of disclosure to learn privately. This delay generates asymmetric information, as the agent is learning while the decision maker is not. This information asymmetry leads to a nontrivial disclosure window and is what allows the decision maker to learn. As disclosure unfolds, information asymmetry is (partially) resolved, and the duration of the disclosure window is determined by the degree of information asymmetry at the beginning of the window, with greater information asymmetry leading to longer windows.

Keywords: dynamic disclosure, strategic timing, dynamic signaling, asymmetric information.

JEL Codes: C73, D82, D83

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1 Introduction

I study a dynamic model of verifiable information disclosure. I consider a disclosure environment in which an agent, who observes a flow of information over time, controls a window of time over which this information is disclosed to a decision maker, but does not control the content of the information being disclosed. The agent chooses when to open this disclosure window and how long to keep it open, taking into account that while disclosure is in progress, he cannot prevent unfavorable information from coming to light.

This disclosure environment is prevalent in many corporate and financial settings. For example, consider a firm developing a prototype. The firm gathers private information during the R&D phase, then decides when to start beta testing and, as testing results arise, when to end it. Following the disclosure of all testing results, the market evaluates the product. In financial disclosure, a firm that privately monitors its operations and finances may opt for a voluntary audit when seeking external investment from private investors or through an Initial Public Offering. The firm chooses when to initiate the audit and, based on the findings, when to conclude it. Upon submission of all audit reports, investors assess the firm's valuation and decide how much capital to allocate.

This disclosure environment prevents the agent from cherry-picking which information to disclose. Instead, the agent affects information by choosing when to open and close the disclosure window. In many settings, information transparency during a disclosure window is a legal requirement, and the timing of disclosure is a crucial strategic component that affects the final outcome. The literature on voluntary disclosure has largely focused on "*what* to disclose," while the study of "*when* to disclose" has received relatively little attention. My paper addresses this gap by studying both *when* and *how long* to disclose, while taking into account that all information arriving during this window of time must be disclosed in full.

Specifically, an agent and a decision maker engage in a dynamic disclosure game that takes place in continuous time over an infinite horizon. The underlying state of the world is either good or bad, initially unknown to both the agent and the decision maker. Over time, the agent privately receives a flow of information about the state in the form of signals that arrive at random times and are conclusive news that the state is bad. The agent chooses a time to start disclosing this information process to the decision maker. While information unfolds, he chooses a time to stop disclosing. To capture the idea that the start and the end of a disclosure process are often restricted by exogenous factors, such as financing and paperwork, I assume that the agent can only start disclosing at any time after getting an opportunity to do so, and disclosure might exogenously terminate at any time after it starts. This means that the decision maker observes the times at which disclosure starts and ends and the signal arrivals (or lack thereof) in between, but not the underlying reasons for why disclosure did not start earlier or why it ended. Given this information, the decision maker takes an action when disclosure ends. While the decision maker prefers an action that matches the state, the agent prefers a higher action regardless of the state. Both players discount future payoff.

This setup poses some analytical challenges. First, this game is effectively a dynamic signaling game with an agent whose private information evolves over time. Second, the agent choosing both when to start and stop disclosing results in two interrelated stopping problems. Third, in addition to private learning, there is also common learning during the disclosure window. These issues lead to a complex inference problem for the decision maker. The standard exponential bandit framework with conclusive bad news helps keep the problem tractable: at any given moment, the agent can be one of only two types: *informed,* if he has observed a conclusive bad signal, or *uninformed,* if he has not. Although the agent's belief still changes over time if he remains uninformed, the agent's type can only change from being uninformed to informed.

I show that there exists an equilibrium of the following form. When choosing when to start disclosing, both the informed and uninformed agent adopt the same strategy. If the agent is patient, he delays the start of disclosure for a fixed amount of time in order to privately learn about the state. Then he starts disclosing as soon as he gets an opportunity. If the agent is impatient, he starts disclosing immediately upon receiving an opportunity right from the beginning of the game. When disclosure starts, the uninformed agent is more optimistic than the decision maker, while the informed agent is more pessimistic. Therefore, the duration of the disclosure window naturally becomes a way for the agent to signal his type: the uninformed agent has stronger incentives to keep disclosure open than the informed agent. In particular, along the history in which no signal arrives and disclosure does not terminate exogenously, the uninformed agent keeps the disclosure window open until the decision maker is convinced that the agent is uninformed. While the uninformed agent strictly prefers to keep the disclosure window open until this time, the informed agent is indifferent and randomizes over waiting times within this disclosure window. The decision maker chooses an action equal to her posterior belief that the state is good at the time disclosure stops.

The equilibrium has several interesting features.

First, the equilibrium captures a novel interaction between the two timing decisions: a later start to disclosure leads to a longer disclosure window. Moreover, the duration of the disclosure window is determined by the degree of information asymmetry between the agent and the decision maker at the beginning of the window. At the beginning of the game, the agent and the decision maker have the same information about the state. Before the agent starts disclosure, he privately learns about the state: he either becomes more optimistic that the state is good, or receives a signal and learns that the state is bad. Because both types of agent adopt the same strategy in choosing when to start disclosure, the decision maker cannot infer anything about the state during this period, while the agent continues to learn. Delaying disclosure thus increases this information asymmetry, and the now-more-optimistic uninformed agent keeps the disclosure window open longer to reduce the difference between his belief that the state is good and that of the decision maker.

Second, because the decision maker's action (her posterior belief) is a martingale of her prior, private information hurts the agent and he would ex ante prefer to avoid becoming more informed than the decision maker. However, because of the stochastic arrivals of disclosure opportunities, information asymmetry is inevitable. Notably, once the agent becomes asymmetrically informed, he may have an incentive to exacerbate this asymmetry. Specifically, the agent can delay the start of disclosure but start immediately once a bad signal arrives. By delaying, the agent privately learns more about the state: if the information is positive, the agent generates more asymmetric information between him and the decision maker, and induces a longer disclosure window; if it is negative, he starts immediately and then stops with positive probability afterwards, which induces a shorter window. As a result, the duration of disclosure becomes positively correlated with the state: longer in the good state, and shorter in the bad one. Delay, however, is costly due to discounting and the additional lag it creates in the continuation game. The agent thus faces a tradeoff between (off-path) private learning and discounting, and chooses to delay only if he is sufficiently patient.

Lastly, the agent's private information is what enables the decision maker to learn about the state. If the agent is known to be the uninformed type at the start of the disclosure window, he has no reason to keep the window open; the uninformed agent uses the disclosure window, the duration and the content, to signal his type and (partially) separate from the informed. But this signaling loses its purpose if the decision maker already knows the agent's type. The decision maker, on the other hand, wants to learn about the state, yet she can only do so if the agent has privately learned something first.

Related Literature

This paper contributes to the literature on voluntary disclosure of verifiable information pioneered by Grossman and Hart (1980), Grossman (1981), and Milgrom (1981). They establish an unraveling result: under certain conditions, all types of agents (or senders) disclose their information and the decision maker (receiver) learns the agent's type. Subsequent work in this literature has shown that the unraveling result fails if disclosure is costly (see Jovanovic, 1982 and Verrecchia, 1983), or if the decision maker is uncertain about the agent's information endowment, that is, whether the agent has information or not (see Dye, 1985 and Jung and Kwon, 1988). I adopt a dynamic version of the second approach.

As mentioned, most of the existing literature has focused on studying "what to disclose" while very few consider "when to disclose."[1] In my paper, not only does the content of the disclosure matter, the timing of disclosure also plays a key role in determining the equilibrium outcome. I discuss some examples that explore the timing of disclosure, all of which are dynamic versions of Dye (1985).

Acharya, DeMarzo, and Kremer (2011) consider a model in which the agent gets one piece of private information and the timing of its disclosure is constrained by the arrival of some public information. Guttman, Kremer, and Skrzypacz (2014) study a two-signal, two-period model where the agent chooses which period to disclose and what signal to disclose. The content of disclosure is completely flexible in Guttman et al. (2014). If the agent waits until the second period to disclose, there is a chance of receiving an additional signal, and he can then choose which signal to disclose. Kremer, Schreiber, and Skrzypacz (2024) model a setting where the underlying state evolves as a random walk rather than remaining fixed. The agent obtains evidence about the state's current value with some probability and chooses whether to disclose it. In their setting, because of the state is evolving, information in the current period is less informative than that in the next. These papers, as well as mine, capture an important consideration in dynamic disclosure problems: the timing of disclosure conveys information in addition to the content of disclosure. In my model, the agent cannot pick and choose the information he discloses. In fact, the agent's inability to control the content of disclosure is one of the key considerations of the agent's timing decisions.

My model features a disclosure environment different from the papers mentioned above. When the agent starts disclosure, he does not disclose a piece of evidence that can be immediately verified. Instead, "disclosure" starts a process where the decision maker can "scrutinize" the information. In this regard, the most closely related paper is Gratton, Holden, and Kolotilin (2018). They study a finite horizon model with a perfectly informed agent who privately observes when she can release a public flow of information, and chooses when to do so. Their model highlights a "credibility vs. scrutiny" tradeoff: the agent signals his (private) information via the time granted to the decision maker to scrutinize (public) information. The continuation game in my model, although studies the agent's decision to stop disclosure not start, features a similar tradeoff where the agent "signals through scrutiny." There are, however, a few key differences. I consider a model where the agent is initially informed and shares the same prior as the decision maker at the beginning of the game, but gradually learns about the state over time. Therefore, information asymmetry is endogenously generated. More importantly, the agent controls both when to start and when to stop disclosure. My analysis focuses on how these two timing decisions interact, and the fact that the agent's private information evolves over time is an important determinant of this interaction.

[1] This was pointed out in, for example, Hirst, Koonce, and Venkataraman (2008), Guttman, Kremer, and Skrzypacz (2014), and Kremer, Schreiber, and Skrzypacz (2024).

On a more technical level, my model results in a dynamic signaling game with changing types. I consider exponential learning with conclusive news (as studied in Keller, Rady, and Cripps, 2005 and Keller and Rady, 2015), which keeps the complications from changing types at bay. Thomas (2019) studies an experimentation problem with reputation concerns where the effect of changing type is more prominent. In addition, I adopt an equilibrium refinement that is in the same spirit as the divinity refinement. Halac and Kremer (2020) also adopts this refinement and applies it to an infinite horizon continuous-time game.

Structure of the paper

I introduce the model in Section 2. In Section 3, I characterize and analyze a class of equilibria. In Section 4, I show that delay in the start of disclosure leads to longer disclosure windows. In Section 5, I study two benchmark models where opportunity arrival or exogenous termination is dropped. I then consider two extensions where the decision maker has more control of the disclosure process: one where she can commit ex ante to a deadline for taking the action, and the other where she chooses the duration of the window. The results from the extensions suggest that the decision maker might not benefit ex post from having more control over the disclosure process. All proofs are relegated to the Appendix or the Online Appendix.

2 Model

Time is continuous and the horizon is infinite. There are two players, an agent (he) and a decision maker (she). There is an unknown state of the world $\theta \in \{0, 1\}$, where $\theta = 1$ indicates the state is good and $\theta = 0$ indicates the state is bad. The players share a common prior that $\theta = 1$, denoted by $\mu \in (0, 1)$.

Information. Over time, the agent receives private signals that have Poisson arrival rate λ^θ, where $\lambda^1 = 0$ and $\lambda^0 = \lambda > 0$. In other words, signals are conclusive that $\theta = 0$. This means the agent's posterior belief that $\theta = 1$ increases in the absence of signals. The agent can receive multiple signals over time, but because signals are conclusive, the agent's belief about the state does not change upon receiving additional signals.

Throughout the game, the agent updates his belief about the state through the realization of the signal process. In particular, let t_s denote the arrival time of the first signal. Define the agent's posterior belief that $\theta = 1$ conditional on no signal arriving by t as $\rho(t) := \Pr(\theta = 1 | t_s > t)$. By Bayes' rule,

$$\rho(t) = \frac{\mu}{\mu + e^{-\lambda t}(1 - \mu)}. \tag{1}$$

This belief $\rho(t)$ is strictly increasing in t. If a signal arrives at t_s, the agent's belief drops down to 0 for all $t \geq t_s$. The agent's belief remains his private information; even though the evolution of the signal process will be publicly observable once the agent starts disclosing, the signal arrivals (or lack thereof) prior to the start of disclosure remains private.

At any t, the agent either has observed a signal and has belief 0 or he has not observed a signal and has belief $\rho(t)$. I say that the agent is *informed* at t if he has observed a signal at or before t, or *uninformed* at t if he has not.[2]

Actions and payoffs. Over time, the agent privately receives an opportunity to start disclosing that has Poisson arrival rate $\alpha > 0$. This process is independent of the state and the signal process. The agent can start disclosing at any time after the arrival of an opportunity.[3] Suppose the first opportunity arrives at time t_o. If the agent starts disclosing at time $t_{\text{start}} \geq t_o$, he commits to disclosing all signal arrivals (or lack thereof) for $t > t_{\text{start}}$. The agent cannot disclose the realization of the signal process for $t \leq t_{\text{start}}$. The decision maker observes t_{start}, the time at which the agent starts disclosing, but not the time at which the opportunity arrives, nor can the agent disclose this information.

Once disclosure starts at t_{start}, the agent chooses a time $t_{\text{stop}} \geq t_{\text{start}}$ to stop disclosing. Also, starting at time t_{start}, an exogenous termination arrives with Poisson arrival rate $\beta > 0$. This process is independent of the state and the signal process. As soon as a termination occurs, the game ends. Suppose a termination occurs at time t_{term}. The decision maker observes the time at which disclosure ends, $\min\{t_{\text{stop}}, t_{\text{term}}\}$, but does not observe whether the end of disclosure is the agent's choice or exogenous.

The decision maker takes an action $a \in \mathbb{R}$ at and only at the time disclosure ends. The decision maker's realized payoff is $1 - (a - \theta)^2$ and the agent's realized payoff is $\kappa + a$, where $\kappa > 0$ is constant.[4] Both players collect payoffs at the time disclosure stops, and discount future payoffs at a common discount rate $r > 0$.

Discussion of Assumptions. The two Poisson arrival processes, opportunity to start disclosing and exogenous termination, capture the idea that the timing of the disclosure window is sometimes restricted by exogenous factors unrelated to the underlying state. Take product testing for example; the company needs to get the relevant paperwork ready, sort out administrative issues, and get financing in order before testing can commence. Similarly, testing may

[2]Note that "time 0" is common knowledge. The decision maker does not know what the agent has learned but does know how much the agent has learned.

[3]I focus on the arrival of the first opportunity. Multiple opportunity arrivals do not matter as long as the agent can start disclosing when the first one arrives.

[4]The constant term $\kappa > 0$ in the agent's payoff captures the idea that delay is costly: if there exists a time $\bar{t}$ such that the decision maker takes a fixed action for any $t \geq \bar{t}$, the agent strictly prefers stopping disclosure at the earliest time $\bar{t}$. The decision maker's payoff has an analogous constant and is normalized to 1.

be cut short unexpectedly due to funding issues or a leadership change, where a new board, who has no interest in developing this product, decides to halt testing prematurely.

From a modeling perspective, these two processes provide noise that prevents the decision maker from getting all of the agent's information upon an observable action. The decision maker observes the times at which disclosure starts and stops, but not the underlying reasons. To be specific, if disclosure has not started, the decision maker cannot determine whether it is because the agent chooses not to start, or because the agent does not have an opportunity to do so. Similarly, if disclosure stops, the decision maker does not know whether it is because the agent chooses to stop, or because disclosure is exogenously terminated.

These two assumptions are analogous to the uncertain information endowment assumption in Dye (1985). Rather than uncertainty about the agent's information endowment, in this dynamic setting, the uncertainty is about when the agent gets the ability to take action. In Section 5.1, I discuss benchmark cases where these two assumptions are dropped.

The Poisson conclusive bad news structure fits well with certain applications: in product testing, it may represent a system failure or critical software bug; in an audit, clear evidence of financial issues. From a modeling perspective, it keeps the analysis tractable—at any point in time, there are only two types of agent, and the equilibrium reduces to characterizing their optimal strategies. While the details may differ under alternative information structures, the core intuition behind the equilibrium is likely to carry over.

Strategies and solution concept. The agent's strategy prescribes when to start and stop disclosing. As is standard, strategies can be described using probability distribution functions. I focus on strategies that satisfy a Markov restriction and introduce the formal definition in the context of Markov strategies.

For a mixed starting strategy, let $\Phi(t|t_o, t_s)$ denote the probability that the agent starts disclosing before or at time t given the opportunity arrives at $t_o < t$ and the first signal at $t_s < t$. Let $\Phi(t|t_o, \varnothing)$ denote the probability that the agent starts disclosure by time t given that the opportunity arrives at $t_o < t$ and no signal has arrived by time t. That is, for each t_o and each t_s, $\Phi(t|t_o, t_s)$ measures, for each t, the probability to start disclosing by time t; for each t_o, $\Phi(t|t_o, \varnothing)$ measures, for each t, the probability to start disclosing by time t. Defining mixed strategies in this way imposes an implicit Markov restriction that specifies that the stopping decision depends only on the time of the first signal arrival, even if the agent may have observed more than one. I impose an additional Markov restriction that specifies

$$\Phi(t|t_o, t_s') = \Phi(t|t_o, t_s'') \text{ for all } t_s', t_s'' < t_o.$$

In words, if the agent is informed by the time he gets the opportunity to disclose, his starting decision does not depend on the time at which he becomes informed.

I impose a similar set of Markov restrictions on the stopping strategy. At the time disclosure starts, denoted by t_{start}, recall that the agent's private history contains the signal arrivals prior to the start of disclosure, and the time of his opportunity arrival. I impose a Markov restriction stating that the agent's stopping strategy only depends on whether he is informed or uninformed prior to the start of disclosure, instead of the entire private history. Because the game ends when an exogenous termination occurs, a stopping strategy only needs to specify the agent's actions in the absence of exogenous termination.

I focus on waiting times w since t_{start}, where waiting time w corresponds to a stopping time of $t_{\text{start}} + w$. Within $[0, w]$, signal arrivals are public. I impose a Markov restriction that specifies that the agent's stopping strategy depends only on the arrival time of the first public signal. Formally, let $w_s \geq 0$ denote the arrival time of the first public signal. Let $G_I(w|t_{\text{start}}, w_s)$ denote the probability that an informed agent stops disclosure by time w, given disclosure started at t_{start} and the first public signal arrives at $w_s < w$. Let $G_I(w|t_{\text{start}}, \varnothing)$ denote the probability that an informed agent stops disclosing by time w, given disclosure started at t_{start} and no public signal has arrived by w. The uninformed agent's probabilities $G_U(w|t_{\text{start}}, w_s)$ and $G_U(w|t_{\text{start}}, \varnothing)$ are defined in the same way. I impose an additional Markov restriction that specifies

$$G_I(w|t_{\text{start}}, \hat{w}) = G_U(w|t_{\text{start}}, \hat{w}).$$

In words, if a public signal arrives at $\hat{w}$, the informed agent and the uninformed agent adopt the same stopping strategies at and after $\hat{w}$.[5]

The decision maker takes an action when and only when disclosure ends. Denote this time by t_{end}. Note that disclosure ends at the minimum of the time the agent stops and the time when termination occurs. Denote the set of public histories at time t_{end} by $\mathcal{H}^{\text{pub}}_{t_{\text{end}}}$ and an element of it $h^{\text{pub}}_{t_{\text{end}}}$. Then $h^{\text{pub}}_{t_{\text{end}}}$ consists of the time at which disclosure starts t_{start}, the time at which disclosure stops t_{end}, and the evolution of the signal process in $[t_{\text{start}}, t_{\text{end}}]$. The decision maker's strategy maps a public history to a real number.

I focus on the set of perfect Bayesian equilibria in which the agent adopts Markov strategies defined above (referred to as equilibrium hereinafter). In addition, I adopt an equilibrium refinement that is in the spirit of the divinity refinement introduced by Banks and Sobel (1987).[6] I discuss how divinity plays a role in more detail when I characterize the equilibrium.

[5] In equilibrium, both types of agent stop disclosing immediately as soon as a public signal arrives.

[6] With a slight abuse of terminology, I refer to this refinement as divinity. The divinity refinement introduced by Banks and Sobel (1987) cannot be applied directly to a continuous time infinite horizon setting.

3 Equilibrium Analysis

I characterize a class of equilibria. I begin by characterizing the decision maker's equilibrium strategy, and devote the rest of the section to characterizing the agent's.

3.1 Decision Maker's Equilibrium Strategy

The decision maker takes an action at and only at the time disclosure ends, t_{end}.[7] As mentioned, a public history at t_{end}, denoted by $h^{\text{pub}}_{t_{\text{end}}}$, contains the time at which disclosure starts, the time at which disclosure ends, and signal arrivals (or lack thereof) in between. Given any public history $h^{\text{pub}}_{t_{\text{end}}}$, the decision maker's maximization problem is

$$\max_{a\in\mathbb{R}} \mathbb{E}_\sigma \left[e^{-rt_{\text{end}}} \left(1-(a-\theta)^2\right) \mid h^{\text{pub}}_{t_{\text{end}}} \right],$$

where the expectation is taken over the agent's strategies, denoted by σ. This implies that in equilibrium, the decision maker's action is equal to her expectation of the state given the public history. Because the state is either 0 or 1, this expectation is equal to her posterior belief that $\theta = 1$. The following lemma summarizes this result.

Lemma 1. *In any equilibrium, at the time disclosure ends t_{end}, given any public history $h^{pub}_{t_{end}}$, the decision maker's action is $a = \mathbb{E}_\sigma[\theta \mid h^{pub}_{t_{end}}] = \Pr(\theta = 1 \mid h^{pub}_{t_{end}})$.*

3.2 Agent's Equilibrium Strategy

To characterize the agent's equilibrium strategies, it is convenient to divide the game into two stages: the initial *starting game* and the continuation *stopping game*.

In the initial starting game, the agent chooses a time to start disclosing. Afterwards, the game moves to the continuation stopping game, which is a game of incomplete information in which at the beginning of the stopping game, the agent is either informed or uninformed. The agent chooses a time to stop disclosing. To keep the analysis in perspective, I first provide a characterization of the equilibrium in Section 3.2, and then analyze the equilibrium in detail and discuss its properties and dynamics in Section 3.3 and Section 3.4.

Equilibrium characterization

Theorem 1 below characterizes a class of equilibria in this game. This class of equilibria features the two types of agent following the same starting strategy and different stopping strategies. Specifically, at the beginning of the game, conditional on having an opportunity,

[7] In particular, the decision maker does not/cannot take an action if disclosure never ends.

both types of agent delay starting until some time $\tilde{\tau} \geq 0$.[8] At and after $\tilde{\tau}$, both types of agent start disclosure upon receiving an opportunity. Once disclosure starts, the uninformed agent waits for a certain amount of time w^* (where w^* depends on the time disclosure starts) to stop disclosure, while the informed agent randomizes over waiting times in $[0, w^*]$ if no signal arrives and termination does not occur.[9] To simplify notation in the continuation game, let $w \in [0, w^*]$ denote the waiting time after disclosure starts. That is, if the agent starts disclosing at t_{start} and waits w to stop, he stops disclosure at the calendar time $t_{\text{start}} + w$.

With a slight abuse of notation, given that disclosure starts at t_{start}, let $G_I(w)$ and $G_U(w)$ denote respectively the informed and uninformed agent's probability of stopping by w given no signal in $[0, w]$ (and no exogenous termination by w). That is, $G_I(w) = G_I(w|t_{\text{start}}, w_s > w)$ and $G_U(w) = G_U(w|t_{\text{start}}, w_s > w)$. The equilibrium can be stated as follows.

Theorem 1.A. *There exists $\tau^* \geq 0$ such that for all $\tilde{\tau} \geq \tau^*$, there is an equilibrium in the starting game of the following form: neither type of agent starts disclosing for $t < \tilde{\tau}$, and both types of agent start disclosing as soon as an opportunity arrives for $t \geq \tilde{\tau}$. That is, for all t_s,*

$$\Phi(t|t_o, t_s) = \begin{cases} 0 & t < \tilde{\tau} \\ 1 & t \geq \max\{\tilde{\tau}, t_o\} \end{cases}.$$

The decision maker's belief that $\theta = 1$ if disclosure starts is 0 for $t < \tilde{\tau}$ (starting disclosure is off-path), and is μ for $t \geq \tilde{\tau}$ (starting disclosure is on-path).

Theorem 1.B. *For each starting time $t \geq \tilde{\tau}$, there exists a unique divine equilibrium of the stopping game. This equilibrium takes the following form: there exists a waiting time $w^* \geq 0$ (where w^* depends on t) such that*

(i) if a signal arrives in $w \in [0, w^*]$, the agent stops disclosing immediately at w;

(ii) if no signal arrives in $[0, w^*]$, the uninformed agent stops at w^* with probability 1:

$$G_U(w) = \begin{cases} 0 & w \in [0, w^*) \\ 1 & w \geq w^* \end{cases}. \tag{2}$$

The informed agent randomizes over $w \in [0, w^*]$. His stopping probability $G_I(w)$ is the unique solution to the following boundary value problem:[10] *for all $w \in [0, w^*]$,*

$$G_I''(w) = \mathscr{G}(G_I(w), G_I'(w), w), \qquad G_I(0) = 0, G_I(w^*) = 1, \text{ and } G_I'(w^*) = 0.$$

[8] Depending on the parameters, $\tilde{\tau}$ can be 0 which means the agent does not delay.

[9] The agent stops immediately upon a signal arrival as the decision maker learns that $\theta = 0$.

[10] Given that disclosure starts at t, in this equilibrium, the uninformed agent's belief that $\theta = 1$ at the

The decision maker's off-path belief that $\theta = 1$ if disclosure stops at $w > w^*$ is the same as the uninformed agent's belief $\rho(t+w)$.

Theorem 1.A characterizes the agent's strategies in the starting game and Theorem 1.B characterizes the agent's strategies in the stopping game. In particular, τ^* is the minimum delay in an equilibrium. Depending on the parameters, either $\tau^* = 0$, meaning the agent starts disclosing as soon as an opportunity arises, or $\tau^* > 0$, meaning the agent must delay the start of disclosure for τ^*, even if an opportunity has arrived before τ^*.

The equilibrium dynamics are best illustrated through the evolution of the decision maker's beliefs. Figure 1 plots an example of the decision maker's on-path belief that $\theta = 1$ conditional on no signal arrivals, in an equilibrium with initial delay till time $\tau^* > 0$.

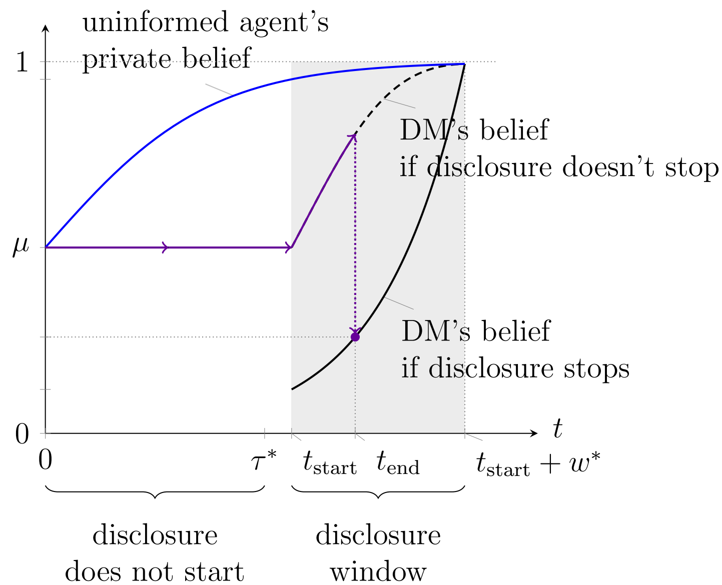

Figure 1: The uninformed agent's and the decision maker's belief evolutions in an equilibrium with initial delay for $\mu = 0.5, r = 0.01, \lambda = 5, \alpha = 1, \beta = 0.5$.

Because signals are conclusive news that $\theta = 0$, the uninformed agent's posterior belief that $\theta = 1$ increases over time in the absence of signals. The decision maker's belief that $\theta = 1$ evolves according to the arrowed path. The two types of agent adopt the same starting

beginning of disclosure t is $\rho(t)$ and the decision maker's is μ. Then $\mathscr{G}$ is given by

$$\begin{aligned}\mathscr{G}(G_I(w), G_I'(w), w) :=& \beta G_I'(w) - \frac{\beta\rho(t)}{\rho(t)-\mu}\left(r(1-\mu) + e^{\lambda w}\mu\left(r\kappa\left(\frac{\mu + e^{-\lambda w}(1-\mu)}{\mu}\right)^2 + r + \lambda\right)\right)\\ &+ r\left(1 + 2\kappa\left(\frac{\mu + e^{-\lambda w}(1-\mu)}{\mu}\right)\right)(\beta G_I(w) - G_I'(w))\\ &- \frac{r\kappa e^{-\lambda w}(\rho(t)-\mu)}{\beta\rho(t)\mu}\left(\beta G_I(w) - G_I'(w)\right)^2.\end{aligned}$$

strategy, so the timing of disclosure, whether it starts before or after τ^*, reveals nothing about the agent's type or the state. As a result, the decision maker's belief remains at her prior until disclosure starts.

Suppose disclosure starts at time t_{start}. Along the history with no signal arrival, disclosure continues for a (maximum) duration of w^*.[11] The shaded area from time t_{start} to $t_{\text{start}} + w^*$ is the *disclosure window* and w^* is its *duration*. During this window, not only is the evolution of the signal process informative of the state, but also stopping of disclosure is bad news about the agent's type. In particular, because only the informed agent intentionally stops during this window, so "not stopping" is good news while "stopping" is bad: the decision maker's belief if disclosure does not stop increases over time, and drops down if disclosure stops.

I now analyze the equilibrium in the two stage games respectively. To better understand the dynamics of this game, it is more intuitive to first solve the continuation stopping game given a starting time, and then work backwards to solve the initial starting game.

3.3 Stopping Game

Fix a public history in which the agent starts disclosing. I impose no assumptions on the structure of the equilibrium in the starting game up to this point other than that the decision maker cannot perfectly infer the agent's type from the time disclosure starts. At the beginning of the stopping game, the agent can be either informed or uninformed. Conceptually, the stopping game can be parameterized by the uninformed agent's (private) belief that $\theta = 1$, denoted by ρ, and the decision maker's belief that $\theta = 1$ conditional on disclosure starting, denoted by η. In this game, the decision maker cannot be more optimistic about the state than the uninformed agent. That is, $\rho \geq \eta$.[12] I characterize the equilibrium conditional on exogenous termination not occurring.[13]

In the stopping game, there are two types of (public) histories: a history along which there is a signal arrival and one where there is not. When a signal arrives at some time, it overrides information transmitted through all other channels. The decision maker's action is 0 at any stopping time after a signal. Because of discounting, the agent strictly prefers stopping immediately at the time of the signal arrival. This is formalized in part (i) of Theorem 1.B.

Along the history with no signal arrival, the decision maker can never be certain of the state, and the agent signals his type through his stopping time. Because of the possibility of

[11] More precisely, w^* is the maximum duration because the agent can choose to stop disclosing prematurely before w^*, or disclosure can terminate exogenously before w^*.

[12] Both ρ and η are determined by the equilibrium starting time t_{start} in the initial starting game. Specifically, $\rho = \rho(t_{\text{start}})$ where $\rho(\cdot)$ is defined in (1). The decision maker's belief is $\eta = \eta(t_{\text{start}})$. (Given the equilibrium starting strategy characterized in Theorem 1.A, $\eta = \eta(t_{\text{start}}) = \mu$ for all t_{start}.)

[13] Recall that the game ends when exogenous termination occurs.

exogenous termination, the informed agent can mimic an uninformed agent whose disclosure process has been exogenously terminated. This gives the uninformed agent incentives to delay stopping in an attempt to separate from the informed type.

To formalize this, I first derive the decision maker's belief when disclosure stops, which is a key element in determining the agent's equilibrium stopping strategy.

Decision maker's (public) belief. While the disclosure window is open, the decision maker updates her belief that $\theta = 1$ through the evolution of the signal process as well as the time disclosure stops. Let $q(w)$ denote the decision maker's posterior belief that $\theta = 1$ if disclosure stops at w. Given disclosure has started, the public history at w consists of the event that no signal arrives in $[0, w]$,[14] and that disclosure stops at w. Let w_s denote the arrival time of the first public signal. Then

$$q(w) := \Pr(\theta = 1 | w_s > w, w_{\text{stop}} = w).$$

To derive this belief, let $F^\theta(w)$ denote the probability that disclosure stops by w conditional on state θ, and $f^\theta(w)$ its density whenever differentiable. Conditional on $\theta = 1$, the agent can only be uninformed. Disclosure stops when either the agent chooses to stop or termination occurs. That is,

$$f^1(w) = G'_U(w)e^{-\beta w} + \beta e^{-\beta w}(1 - G_U(w)). \tag{3}$$

To derive the rate at which disclosure stops at w conditional on $\theta = 0$, it is useful to first define the probability that the agent is informed at the beginning of the continuation game conditional on $\theta = 0$. Denote this probability by γ, then $\eta = \rho(1 - \gamma(1 - \eta))$. By the same logic as the $\theta = 1$ case, conditional on $\theta = 0$,

$$f^0(w) = (1 - \gamma)f^1(w) + \gamma \left[G'_I(w)e^{-\beta w} + \beta e^{-\beta w}(1 - G_I(w))\right]. \tag{4}$$

By Bayes' rule,

$$q(w) = \frac{f^1(w)\eta}{f^1(w)\eta + e^{-\lambda w} f^0(w)(1 - \eta)}. \tag{5}$$

Agent's expected payoff. Given $q(w)$, the informed agent's expected payoff from waiting w to stop disclosing consists of three cases. (1) A signal arrives before a termination occurs during the waiting time. The agent stops immediately and the decision maker takes action 0. (2) A termination occurs at some s before a signal arrives. The decision maker takes action $q(s)$, where $q(\cdot)$ is given by (5). (3) No signal arrives and no termination occurs. The decision

[14] Recall that $w \geq 0$ is the waiting time after disclosure starts, not the calendar time.

maker takes action $q(w)$ at w.[15] Denote by $V(w)$ the informed agent's expected payoff from waiting w, and $U(w)$ the uninformed agent's. Then

$$V(w) = \int_0^w e^{-rs}\lambda e^{-\lambda s}e^{-\beta s}\kappa \mathrm{d}s + \int_0^w e^{-rs}e^{-\lambda s}\beta e^{-\beta s}(\kappa + q(s))\mathrm{d}s + e^{-rw}e^{-\lambda w}e^{-\beta w}(\kappa + q(w))$$

and

$$U(w) = (1-\rho)V(w) + \rho\left(\int_0^w e^{-rs}\beta e^{-\beta s}(\kappa + q(s))\mathrm{d}s + e^{-rw}e^{-\beta w}(\kappa + q(w))\right).$$

It can be verified that if $V(w)$ is (weakly) increasing in w, then $U(w)$ is strictly increasing in w. Intuitively, conditional on $\theta = 0$, the longer disclosure stays open, the more likely a signal is to arrive. Since the uninformed agent is more optimistic, if the informed agent is (weakly) willing to wait, the uninformed agent strictly prefers to wait.

Using this property, I show existence of an equilibrium of the following form. There exists $w^* \geq 0$ such that the informed agent randomizes over waiting times in $[0, w^*]$, while the uninformed agent strictly prefers to wait w^*. Both types stop with probability 1 by w^*. I then show this is the unique equilibrium that survives the divinity refinement.

3.3.1 Equilibrium Dynamics

As previously noted, the dynamics of the equilibrium are best understood through the evolution of the decision maker's beliefs. In equilibrium, the informed agent's randomization over waiting times feeds into the decision maker's belief evolution which in turn keeps the informed agent's expected payoff from stopping constant over time. Specifically, $V'(w) = 0$ implies

$$q'(w) = r\kappa + (r+\lambda)q(w). \tag{6}$$

While the window is open, the informed agent faces two exponential forces—discounting over time and the stochastic arrival of signals. The indifference condition (6) therefore requires the decision maker's belief to rise at an exponential rate to offset these opposing pressures.[16]

In this game, the decision maker's belief that $\theta = 1$ is bounded above by the uninformed agent's at any point in time. This suggests $q(w)$ cannot exponentially increase indefinitely and disclosure must stop before this belief exceeds the uninformed agent's. The following lemma takes this observation one step further and shows that if disclosure stops at the end of the disclosure window, the decision maker infers that the agent must be uninformed, and

[15] The event that both a signal and termination arrive in $[w, w + \mathrm{d}w)$ is of order $(\mathrm{d}w)^2$ and can be neglected.

[16] Specifically, the solution to the first-order differential equation (6) is $q(w) = me^{(r+\lambda)w} - \kappa r/(r+\lambda)$ with constant $m > 0$ to be determined in equilibrium.

therefore, her posterior belief that $\theta = 1$ must equal the uninformed agent's.

Lemma 2. *If the informed agent's strategy is atomless with support* $[0, w^*]$*, then*

$$q(w^*) = \frac{\rho}{\rho + e^{-\lambda w^*}(1-\rho)}. \tag{7}$$

Moreover, $w^* < \infty$*.*

Intuitively, at the end of the disclosure window, both types of agent stop disclosing with probability 1. Moreover, the informed agent stops continuously throughout. Thus, there is no mass of the informed type left at w , and stopping can only come from the uninformed agent. As will be discussed later, this structure where the informed agent stops continuously over the entire disclosure window is the unique equilibrium that survives the divinity refinement.

To further understand the equilibrium dynamics, consider the decision maker's belief before disclosure stops, denoted by $q^C(w)$. That is,

$$q^C(w) \coloneqq \Pr(\theta = 1 | w_s > w, w_{\text{stop}} > w).$$

Because only the informed agent voluntarily stops during the disclosure window, $q^C(w)$ is increasing in w and $q^C(w) > q(w)$ for all $w < w^*$.

In summary, from the decision maker's perspective, stopping prematurely before w^* is bad news—whenever disclosure stops, the decision maker's belief drops from $q^C(w)$ to $q(w)$. However, later premature stopping is good news—a longer duration without a signal arrival induces a higher belief that the state is good, and later stopping implies there's a higher chance that the agent is uninformed to begin with. Figure 2 illustrates the dynamics by plotting the decision maker's beliefs as functions of waiting time w.

While the evolution of the decision maker's belief $q(w)$ illustrates the equilibrium dynamics, it does not pin down the equilibrium strategies $G_I(w)$—some properties of the agent's strategies get lost during their translation to beliefs. In the strategy space, $G_I(w)$ is continuous at any $w \in [0, w^*]$, in particular, at $w = 0$, so $G_I(0) = 0$. The informed agent stops with probability 1 by w^* means $G_I(w^*) = 1$. Together with Lemma 2, they imply $G_I'(w^*) = 0$. Moreover, equation (6) defines a second-order differential equation in G_I. All together, they form the boundary value problem stated in part (ii) of Theorem 1.B. By a shooting argument, this boundary value problem has a unique solution. The left panel of Figure 3 plots this solution, $G_I(w)$, as well as the uninformed stopping strategy, $G_U(w)$.

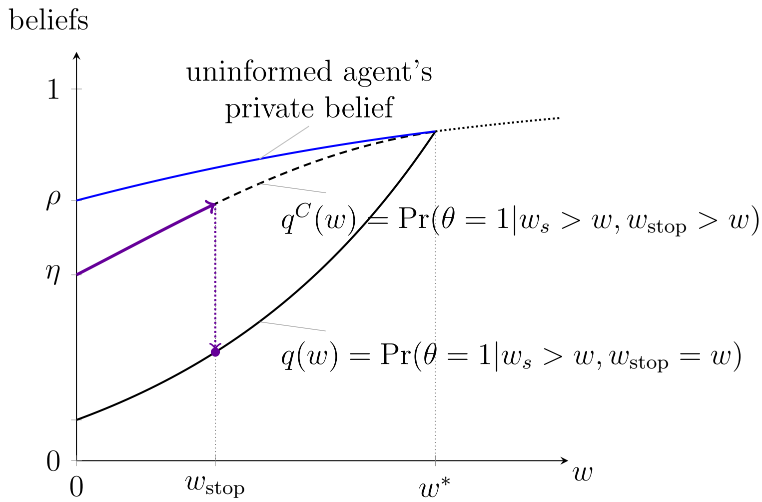

Figure 2: Belief evolutions for $\rho = 0.7$, $\eta = \mu = 0.5, \lambda = 3, r = 0.5, \alpha = 1, \beta = 0.5$, and $\kappa = 1$.

3.3.2 Divinity and Equilibrium Uniqueness

Given a fixed equilibrium, divinity should specify that the decision maker's off-path belief assigns zero weight to the agent being the type that has less incentive to deviate. In this game, continuing disclosure after w^* is off the equilibrium path. Intuitively, for any given belief of the decision maker, the uninformed agent always has a stronger incentive than the informed agent to keep disclosure open. Therefore, the decision maker's off-path belief should assign zero weight to the agent being informed.

This means that the uninformed agent can always "prove" that he is uninformed by deviating this off-path play. If the uninformed agent were to ever stop on the equilibrium path, it must be that whenever the he chooses to stop, the decision maker's belief upon stopping is that the agent is uninformed. Once the decision maker believes that the agent is uninformed, she and agent have the same belief about the state. Because their common posterior beliefs are a martingale of their current belief, stopping at any $w \geq w^*$ induces the same expected posterior belief. Given discounting, the uninformed agent stops at w^*.

The above argument shows that in a divine equilibrium, the decision maker's belief that $\theta = 1$ at w^* must be equal to the uninformed agent's. In addition, the decision maker's belief must also be continuous at w^*: any jump at w^* will result in a profitable deviation to stopping at w^* for the informed agent. These conditions pin down the boundary conditions in the boundary value problem stated in Theorem 1.B (ii), giving rise to equilibrium uniqueness.

Without divinity, one can construct equilibria where disclosure stops after any waiting time $\overline{w} \leq w^*$ by imposing a punishing belief off the equilibrium path. More precisely, for each $\overline{w} \leq w^*$, one can construct an equilibrium where the informed agent randomizes over waiting times in $[0, \overline{w}]$, while the uninformed agent waits $\overline{w}$. Both types stop with probability 1 by

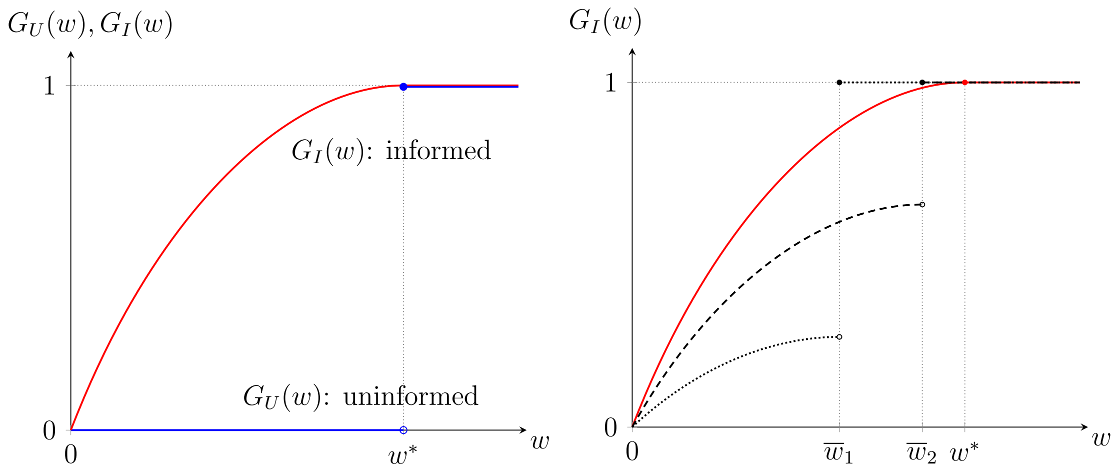

Figure 3: Parameters are $\rho = 0.7, \eta = 0.5, \lambda = 3, r = 0.5, \alpha = 1,$ and $\beta = 0.5$. The left panel plots the uninformed and informed agent's stopping strategies in a divine equilibrium and the right panel plots two examples of the informed agent's stopping strategies in a non-divine equilibrium ($\overline{w}_1$ and $\overline{w}_2$), alongside the divine equilibrium for comparison.

$\overline{w}$. Off the equilibrium path, if disclosure stops at any $w > \overline{w}$, the decision maker believes the agent is informed. In these equilibria, the informed agent stops with a strictly positive probability at $\overline{w}$, in other words, there is a jump in $G_I(\cdot)$ at $\overline{w}$. The size of the jump must make the informed agent indifferent between stopping at $\overline{w}$ and any earlier point. The right panel of Figure 3 plots the informed agent's strategy in two such equilibria, as well as the divine equilibrium for comparison.

3.3.3 Role of Information Asymmetry

Because the agent might be informed before starting disclosure, at the time of disclosure, the decision maker's belief is lower than the uninformed agent's. As it turns out, it is precisely this information asymmetry that enables information transmission in the stopping game.

To see this, suppose the agent is uninformed and the decision maker knows it. Therefore, the decision maker and agent have the same belief about the state. Because their common posterior beliefs are a martingale, stopping at any time would induce the same expected action. Discounting drives the agent to stop disclosing immediately; the decision maker takes an action that is equal to the common belief. Hence, it is the possibility that the agent might be informed forces the uninformed agent to keep the disclosure window open, hoping that the lack of signals corroborates that he is uninformed, enabling information transmission to the decision maker. The following lemma formalizes this result.

Proposition 1. *$w^* > 0$ if and only if $\rho > \eta$.*

In a sense, the agent does not care about what he knows but what the decision maker believes. The uninformed agent is more optimistic than the decision maker, and his incentive to delay stopping and let information flow is not driven by learning for himself. Instead, he delays so that the decision maker can learn and become as optimistic as he is.

A key comparative static result. As analyzed above, the duration of the disclosure window can be interpreted as the amount of time needed to eliminate the information asymmetry between the agent and the decision maker at the start of disclosure. It is intuitive that the larger this information asymmetry is, the longer it takes to weed it out. This suggests that the duration of the disclosure window should increase in the degree of information asymmetry. The following result formalizes this intuition.

Lemma 3. *The equilibrium waiting time w^* is increasing in ρ and decreasing in η.*

Although intuitive, w^* is part of the solution to a boundary value problem, so proving Lemma 3 is not straightforward. Nevertheless, this result is crucial in analyzing the starting game in Section 3.4, and deriving one of the main insights of the paper in Section 4. The analysis so far has shown that the degree of information asymmetry at the beginning of the disclosure window impacts the duration of the disclosure window. Section 4 concerns how this degree of information asymmetry is affected by the time disclosure starts. On that note, I now turn to the agent's optimal time to start disclosure.

3.4 Starting Game

In the initial starting game, the agent chooses when to start disclosure, anticipating that this choice determines the duration he will later choose, as characterized by the unique divine equilibrium in the continuation stopping game.

Depending on the parameters, an equilibrium in the starting game can feature either immediate disclosure or delayed disclosure. In an *immediate disclosure equilibrium*, both types of agent start disclosing as soon as he gets the opportunity regardless of the date of that opportunity. In a *delayed disclosure equilibrium*, there is a fixed period at the beginning of the game, $[0, \tilde{\tau})$, during which neither type starts disclosing even if an opportunity arrives, and starts disclosing immediately at and after $\tilde{\tau}$. That is, the agent discloses at the larger of $\tilde{\tau}$ and when she gets the opportunity.

I make two remarks. First, delay is sometimes necessary. While one can always impose an initial delay by assigning a punishing belief to early disclosure, in some cases, delay can arise endogenously as the agent would deviate to waiting if we try to construct an equilibrium without delay. Second, the agent's incentive to delay is subtle. Ex ante, the agent and the decision maker have the same belief about the state. Given the martingale property of posterior beliefs and discounting, delaying disclosure is always ex ante suboptimal for the agent. The agent gains nothing from learning about the state, and would start disclosing immediately (and then stop immediately) at time 0 if possible.

The equilibrium therefore highlights a key insight: once the agent (exogenously) becomes asymmetrically informed, he has an incentive to (endogenously) become even more so—an incentive he does not have when he has no private information.

To see this, suppose both types of agent start disclosing immediately. Then the decision maker's belief about the state when disclosure starts equals her prior, as she cannot infer the agent's type from the start time. Given this belief, the uninformed agent can profitably deviate to delaying the start of disclosure for some time, but starting immediately if a signal arrives. At time t, if the agent waits till $t + \mathrm{d}t$ to start, he privately learns about the state: if no signal arrives in $[t, t + \mathrm{d}t)$, the agent becomes more optimistic and generates greater information asymmetry, so he gets to keep the disclosure window open longer in the continuation game; if a signal arrives, the agent starts disclosure immediately, and then stops optimally with positive probability afterwards, inducing a shorter window in expectation. As a result, the duration of disclosure becomes positively correlated with the state.

To see why this correlation is beneficial, note that if there is no discounting, in the bad state, shorter disclosure is better due to the risk of a signal arriving. In the good state, longer disclosure is better: with no risk of signals or discounting, the later the agent stops, the higher the decision maker's action. Therefore, the agent benefits from generating a higher correlation between the state and the duration: he induces a longer window when it is favorable to him (in the good state), and a shorter window when it is not (in the bad state). However, delay postpones the final action and is costly due to discounting. The agent thus faces a tradeoff between (off-path) private learning and discounting, and would benefit from delay when (1) he is patient so the delayed payoff is not too dissipated, and (2) he is early in the game, when he is most uncertain about the state so learning is most valuable.

I formalize these results below. I first provide a sufficient condition under which immediate disclosure is an equilibrium. This condition formalizes the intuition above: if discounting is high relative to information arrival, delay is not worthwhile. I then show if discounting is low, immediate disclosure cannot be an equilibrium. In such cases, I construct an equilibrium with an initial delay. Finally, using a numerical example, I demonstrate that there exist parameters where the equilibrium with the shortest initial delay survives the divinity refinement.

To simplify the exposition, I impose two parametric assumptions: (i) the prior that $\theta = 1$ is $\mu \geq 1/2$ and (ii) $\kappa = 0$. The first assumption does not affect the stopping game but simplifies the starting game and delivers a clean intuition. For the second assumption, $\kappa > 0$ in the continuation game ensures that the agent prefers stopping immediately (to delay) upon the arrival of a signal and at the end of the disclosure window. The equilibrium in the continuation game goes through if we assume $\kappa = 0$ and the agent stops at the earlier time when indifferent.

3.4.1 Immediate Disclosure Equilibrium

The following proposition provides sufficient conditions under which it is an equilibrium for both types of the agent to start disclosing as soon as they get an opportunity.

Proposition 2. *If $r/\lambda \geq 1 - \mu$, then immediate disclosure is an equilibrium.*

Rewrite the condition as $\lambda(1-\mu)\mu \leq r\mu$. This condition compares the marginal benefit and marginal cost of an infinitesimal delay for an uninformed agent in the best-case scenario: whenever disclosure starts, it stops immediately, and the decision maker believes that the agent is uninformed. The decision maker takes the highest possible action and the agent incurs no waiting to induce it.[17] The left-hand side, $\lambda(1-\mu)\mu$, is the marginal benefit of delay in this best-case scenario. At time 0, with an expected rate of $\lambda(1-\mu)$, a signal arrives. By delaying, the agent privately learns that the state is $\theta = 0$, then immediately stops and gets payoff μ. Had he disclosed right away, the decision maker would have seen the signal, taken action 0, and the agent would have received a payoff of 0. The gain from delaying is thus μ. The right-hand-side, $r\mu$, is the marginal cost of delay. Because there is no waiting in the continuation game, the cost purely comes from discounting.

The condition says that, in the best-case scenario where the agent receives the highest possible payoff without waiting, discounting is so large relative to information that the agent has no incentive to delay. Thus, in the case where the agent does wait in the continuation game and have to wait even longer if he starts later, he would be even less inclined to delay.

3.4.2 Delayed Disclosure Equilibrium

I now show that for a small r, immediate disclosure cannot be an equilibrium. I then characterize the minimum delay τ^* in a delayed disclosure equilibrium.

Suppose the agent has an opportunity to start disclosing at time 0. Let $Y(t)$ denote his expected payoff from delaying until t if he remains uninformed at t, and starting immediately at $s < t$ if he become informed at s. Let $V^*(t)$ and $U^*(t)$ denote the informed and uninformed agent's equilibrium payoff in the continuation game if disclosure starts at t. Then from the perspective of time 0,

$$Y(t) = (1-\mu)\int_0^t \lambda e^{-\lambda s} e^{-rs} V^*(s)\mathrm{d}s + \left((1-\mu)e^{-\lambda t} + \mu\right) e^{-rt} U^*(t).$$

[17] The highest possible action is subject to the decision maker's optimality: the decision maker's action is equal to her posterior belief that $\theta = 1$. In this best-case scenario, this belief is equal to the uninformed agent's belief which is the highest possible belief any player can have.

It suffices to focus on the agent who has the opportunity to start at time 0. Lemma 20 in the Appendix proves that if the agent gets the opportunity at $t_0 > 0$, his incentive at t_0 is the same as the agent who has the opportunity before t_0.

For immediate disclosure to be an equilibrium, $Y(t)$ must be decreasing for all $t \geq 0$. If $Y(t)$ is increasing at point in time, an agent who gets the opportunity at that time would prefer to delay, contradicting the definition of the immediate disclosure equilibrium. The next lemma shows that for each $t > 0$, $Y'(t)$ is increasing when r is small (though "how small" might depend on t). It follows that immediate disclosure is not an equilibrium for a small r.

Lemma 4. *For any $t > 0$, $Y'(t)$ is continuous in r and $\lim_{r\to 0} Y'(t) > 0$.*

This implies for each $t > 0$, there exists at least an ε-neighborhood (ε might depend on t) around $r = 0$ such that $Y(\cdot)$ is increasing at t for all r in that neighborhood. Thus, for a sufficiently small r, immediate disclosure is not an equilibrium, as there exists t where the agent would prefer to delay if he gets the opportunity then.

Lemma 5. *There exists $\bar{t}$ such that $Y'(t) \leq 0$ for all $t \geq \bar{t}$.*

This result implies that the agent will eventually want to start disclosing immediately. Under Poisson learning, the marginal value of learning diminishes over time. Learning is most valuable early in the game when the agent is most uncertain about the state. Eventually, the benefit from private learning can no longer outweigh the cost of delay due to discounting.

Define τ^* as the earliest time after which delaying is no longer beneficial:

$$\tau^* := \inf\{\tilde{t} \geq 0 : Y'(t) \leq 0 \text{ for all } t \geq \tilde{t}\}.$$

That is, after τ^*, the uninformed agent optimally starts disclosing immediately. For immediate disclosure equilibrium, τ^* is defined the same way and equals zero. This completes the equilibrium characterization of the starting game (Theorem 1.A).

3.4.3 Divinity and Equilibrium Multiplicity

The immediate disclosure equilibrium survives the divinity refinement trivially because there are no off-path actions. In a delayed disclosure equilibrium, the agent starts disclosing only after some $\tilde{\tau} > 0$, so starting disclosure before $\tilde{\tau}$ is off the equilibrium path. In constructing this equilibrium, the decision maker's off-path belief upon early disclosure is set to $\theta = 0$. Is this off-path belief consistent with the divinity refinement?

As discussed in the stopping game in Section 3.3, the divinity refinement specifies that the decision maker's off-path belief should assign zero weight to the type of agent that has less incentive to deviate, in the sense that the set of responses by the decision maker for which the deviation is profitable is strictly smaller. If the "zero off-path belief" is consistent with divinity, at each $t < \tau^*$, for any decision maker's belief $\eta \in [0, \rho(t)]$ such that the uninformed agent finds it profitable to deviate to starting at t, the informed agent must find it strictly profitable to deviate. In other words, the set of decision maker's beliefs that induce a profitable deviation for the uninformed agent must be strictly contained in the set for the informed agent.

Whether the divinity condition is satisfied depends on the parameters. In a numerical example illustrated below, the equilibrium with the minimal delay τ^* is divine. The left panel of Figure 4 illustrates that this minimum-delay equilibrium satisfies divinity, while the right panel illustrates an equilibrium with a longer delay $\tilde{\tau} > \tau^*$ that does not. The red (solid) area plots the set of decision maker's belief η such that the informed agent finds it profitable to deviate, and the blue (patterned) area is where the uninformed agent finds profitable to deviate. To satisfy divinity, the blue set needs to be entirely contained in the red.

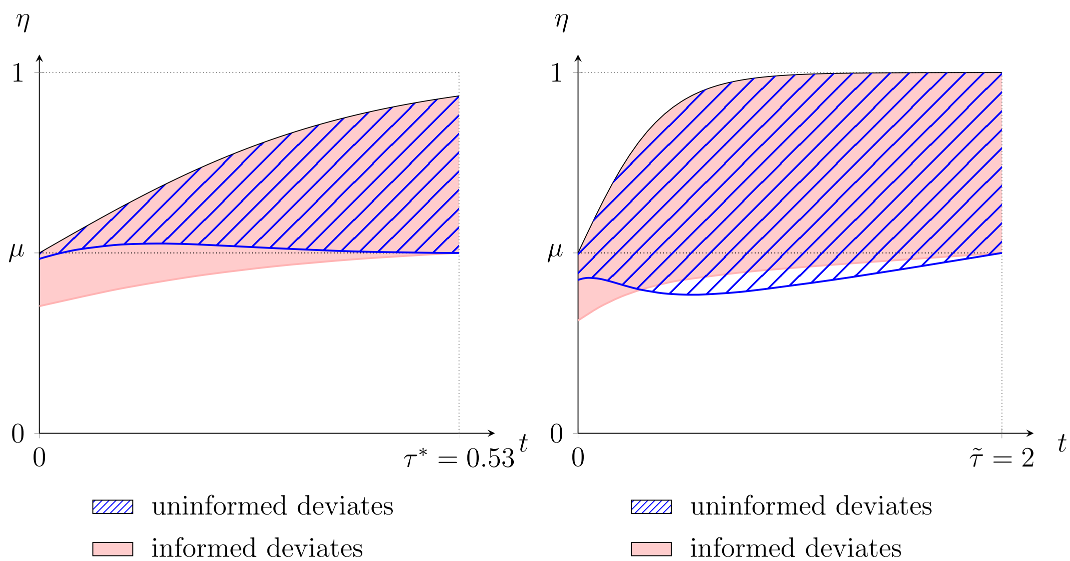

Figure 4: Examples where a delayed disclosure equilibrium is divine (left panel, $\tilde{\tau} = \tau^*$) and is not (right panel, $\tilde{\tau} > \tau^*$) for $\mu = 0.5, r = 0.01, \lambda = 5, \alpha = 1, \beta = 0.5$, and $\kappa = 1$.

In general, the initial starting game has multiple equilibria. Within the class of pooling equilibria characterized here, there is a continuum of equilibria, where each $\tilde{\tau} \geq \tau^*$ can be sustained as an equilibrium. The numerical examples above show that divinity can indeed eliminate some equilibria, but in general, it cannot reduce the equilibrium set to a singleton.

4 Duration of Disclosure Windows

The equilibrium features a novel dynamic between the start disclosing time and the stop disclosing time: delay in the start of disclosure leads to a longer disclosure window. Proposition 3 formalizes this result and Figure 5 illustrates.

Proposition 3. *The agent's waiting time $w^*(t)$ is increasing in the disclosure starting time t. Moreover, there exists $\overline{w}^* < \infty$ such that $\lim_{t\to\infty} w^*(t) = \overline{w}^*$.*

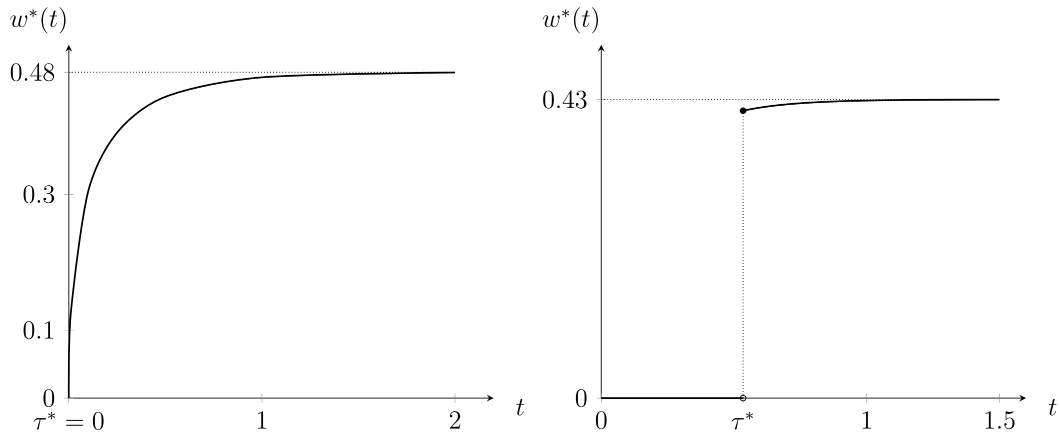

Figure 5: Agent's waiting time $w^*(t)$ as a function of the disclosure starting time t. Left panel: $\tau^* = 0$; $\mu = 0.5, r = 0.5, \lambda = 3, \alpha = 1, \beta = 0.5$, and $\kappa = 1$. Right panel: $\tau^* = 0.53$; $\mu = 0.5, r = 0.01, \lambda = 5, \alpha = 1, \beta = 0.5$, and $\kappa = 1$.

The agent and the decision maker start off with the same information about the state, but as the start of disclosure is delayed, they become increasingly asymmetrically informed. During this delay, the agent privately learns about the state through the signal process: he becomes more optimistic in the absence of signals or becomes fully informed if a signal arrives. The decision maker, by contrast, receives no information—she does not observe the signal process and cannot infer anything from the agent's behavior, as both types follow the same starting strategy. Thus, on the equilibrium path, her belief that $\theta = 1$ remains constant at the prior. The longer disclosure is delayed, the greater the information asymmetry, and the longer the uninformed agent must keep the disclosure window open to eliminate it.

If disclosure starts sufficiently late, the continuation game approaches a benchmark case in which the agent is fully informed. Let $\overline{w}^*$ denote the optimal waiting time in this benchmark case. The waiting time $w^*(t)$ then converges to $\overline{w}^*$ as t increases. In the Online Appendix, I present this benchmark model and characterize $\overline{w}^*$.

5 Benchmarks and Extensions

5.1 Exogenous Termination and Starting Opportunity

Suppose there is no exogenous termination. There exists an equilibrium where both types of agent start disclosing as soon as they get an opportunity and adopt a stopping strategy that fully reveals their type: for each starting time $t \geq 0$, there is a unique waiting time $w^{\dagger}(t) > 0$ such that the uninformed agent stops at $w^{\dagger}(t)$ if no signal arrives, while the informed agent stops at 0. The optimal waiting time $w^{\dagger}(t)$ is such that the informed agent is indifferent between stopping at 0 and at $w^{\dagger}(t)$. Thus, stopping immediately and stopping at $w^{\dagger}(t)$ respectively reveal that the agent is informed and uninformed. The decision maker's (off-path) belief if disclosure stops during $(0, w^{\dagger}(t))$ is that the agent is informed and $\theta = 0$.

For starting time $t > 0$, this is the unique divine equilibrium. For $t = 0$, there exists another divine equilibrium that is pooling: the agent stops disclosing immediately at $t = 0$, and the decision maker's belief that $\theta = 1$ is equal to the common prior.

For $t > 0$, as the exogenous termination arrival rate β converges to 0, the equilibrium of the main model converges pointwise to the unique (separating) equilibrium characterized by $w^{\dagger}(t)$. For $t = 0$, which of the two equilibria it converges to depends on the order of limits. Take the limit $\beta \to 0$ and then $t \to 0$, the equilibrium of the main model converges (pointwise) to the separating equilibrium; the reversing order yields the pooling equilibrium.

It remains true that a later start leads to a longer window. The duration of the disclosure window only depends on the uninformed agent's belief at the beginning of the window, and this belief increases the later disclosure starts.

This means that the disclosure window only depends on the calendar time at which it starts. As a result, both types of agent start disclosing as soon as they get an opportunity.

Suppose the agent can start disclosing at any point in time. Because of the martingale property of posterior belief and discounting, the agent would not want to delay. The unique equilibrium thus features no strategic interactions and no information transmission: the agent starts disclosing at time 0 and immediately stops disclosing at time 0. The decision maker takes an action that is equal to the common prior.

5.2 More Control to the Decision Maker

A natural question is what would happen if the decision maker has more control over the disclosure process beyond taking the final action. I consider two such interventions: allowing the decision maker to commit ex ante to a (history-contingent) deadline for taking the final action, and allowing the decision maker to determine the duration of the disclosure window instead of the agent. The results suggest that even though these mechanisms give the decision maker greater procedural control, they do not necessarily make her better off ex post.

Committing to a deadline. Suppose the decision maker can commit ex ante to a deadline for taking the final action. Specifically, before the game begins, she commits to a rule that assigns a decision time for each possible disclosure start time: if no signal arrives and disclosure has not stopped by this deadline, she takes an action. If a signal arrives before the deadline, she takes action 0 immediately. The agent can stop disclosure before the deadline.

The deadline only binds if it is shorter than the equilibrium stopping time w^*. In that case, the game admits an equilibrium that is outcome-equivalent to a non-divine equilibrium of the main model in which disclosure ends at some fixed $\overline{w} < w^*$. That is, committing to a deadline has the same effect as exogenously imposing a stopping time $\overline{w}$ and assigning a off-path belief that the agent is informed if disclosure continues past $\overline{w}$.

Choosing disclosure duration. Suppose the decision maker, rather than the agent, chooses the stopping time of disclosure. As in the main model, she can stop and take an action only if disclosure has started at some prior time. The rest of the setup remains unchanged.[18]

In this model, the decision maker's problem is Markov in her belief. She either stops when she sees a signal and thus learns the state is 0, or she waits until her belief reaches a threshold $\overline{\eta}$ that is independent of the time at which disclosure started. As before, denote the decision maker's belief at the beginning of the continuation game by η. The following proposition formalizes this intuition. (The proof is relegated to the Online Appendix.)

Proposition 4. *There exists $\Delta > 0$ such that*
(i) if $r/\lambda \geq \Delta$, the decision maker stops immediately for all $\eta \in (0,1)$;
(ii) if $r/\lambda < \Delta$, there exists $\underline{\eta}$ and $\overline{\eta}$ with $0 < \underline{\eta} < \overline{\eta} < 1$ such that if $\eta < \underline{\eta}$ or $\eta \geq \overline{\eta}$, the decision maker stops immediately. If $\eta \in [\underline{\eta}, \overline{\eta})$, the decision maker either stops at the first time her posterior belief is equal to 0 or the first time her posterior belief is equal to $\overline{\eta}$.

Because the agent does not choose the stopping time, there is no value in private learning in the starting stage. So both types of agent start disclosing as soon as an opportunity arrives.

The decision maker sometimes is better off letting the agent choose the duration of disclosure. Intuitively, the agent's private information about the state accumulates before the start of disclosure. In the case where the decision maker chooses the stopping time, the decision maker has no way of extracting the agent's private information. On the other hand, if the agent chooses the stopping time, the stopping time signals the agent's private information. Therefore, if the disclosure starts late when the agent has already accumulated a significant amount of information, the decision maker benefits from letting the agent choose the stopping time and learn at a faster rate.

[18] In particular, I maintain the assumption that disclosure can exogenously terminate at some random time after it starts. This assumption plays no role other than to keep the model comparable with the main model.

A Appendix

A.1 Proofs for Section 3

A.1.1 Proof of Lemma 1

Given any public history h_T^{pub} at T, denote $\Pr(\theta = 1|h_T^{\text{pub}}) = q(T)$. The decision maker's problem is $\max_{a\in\mathbb{R}} e^{-rT}\left[\left(1-(a-0)^2\right)(1-q(T)) + \left(1-(a-1)^2\right)q(T)\right]$. Setting the derivative with respect to a to 0 results in $a = q(T)$. It can be readily verified by checking the second derivative that this payoff function is maximized at $a = q(T)$.

A.1.2 Proof of Theorem 1.B (i)

Suppose a signal arrives at $\hat{w} \geq 0$. Then both the agent and the decision maker's belief is zero for all $w \geq \hat{w}$. The decision maker takes action 0 if disclosure stops at any $w \geq \hat{w}$. The agent's expected payoff from stopping at any $w \geq \hat{w}$ is $e^{-r(w-\hat{w})}\kappa$, which is maximized at $\hat{w}$.

A.1.3 Proof of Theorem 1.B (ii)

A.1.3 (I) Preliminaries

First, I introduce a useful notation for the uninformed agent's belief in the continuation game.

Suppose disclosure starts at t_{start}. At the beginning of the continuation stopping game, the uninformed agent's belief is $\rho(t_{\text{start}})$, where $\rho(\cdot)$ is given by (1). To simplify notation, let $\rho = \rho(t_{\text{start}})$ and denote by $p(w)$ the uninformed agent's private belief that $\theta = 1$ calculated from the perspective of t_{start}. In other words, $p(w) = \rho(t_{\text{start}} + w) = \rho/\left(\rho + e^{-\lambda w}(1-\rho)\right)$.

Next, I write $F^1(w)$ and $F^0(w)$ as functions of the agent's strategies $G_U(w)$ and $G_I(w)$. Recall that $F^\theta(w)$ is the probability that disclosure stops by w given state θ. So

$$\begin{aligned}
F^1(w) =& 1 - \Pr(w_{\text{stop}} > w|\theta = 1, \text{uninformed}) \\
F^0(w) =& 1 - \Pr(w_{\text{stop}} > w|\theta = 0, \text{informed}) \Pr(\text{informed}|\theta = 0) \\
&- \Pr(w_{\text{stop}} > w|\theta = 0, \text{uninformed}) \Pr(\text{uninformed}|\theta = 0).
\end{aligned}$$

Disclosure stops either because the agent chooses to stop or it exogenously terminates. Because exogenous termination is independent of θ, so

$$\begin{aligned}
\Pr(w_{\text{stop}} > w|\theta = 1, \text{uninformed}) &= e^{-\beta w}\left(1 - G_U(w)\right) \\
\Pr(w_{\text{stop}} > w|\theta = 0, \text{uninformed}) &= e^{-\beta w}\left(1 - G_U(w)\right) \\
\Pr(w_{\text{stop}} > w|\theta = 0, \text{informed}) &= e^{-\beta w}\left(1 - G_I(w)\right).
\end{aligned}$$

By definition, $\Pr(\text{informed}|\theta = 0) = \gamma$. After some simplifying,

$$F^1(w) = 1 - e^{-\beta w}\left(1 - G_U(w)\right) \tag{8}$$

$$F^0(w) = 1 - e^{-\beta w}\left(1 - (1-\gamma)G_U(w) - \gamma G_I(w)\right). \tag{9}$$

Finally, I derive properties of $q(w)$ when either G_U or G_I is discontinuous.

Suppose at some $\hat{w} \geq 0$, either G_U or G_I is discontinuous. Then $q(\hat{w})$ can be written as

$$q(\hat{w}) = \left(1 + e^{-\lambda\hat{w}} \lim_{\varepsilon\to 0} \frac{F^0(\hat{w}) - F^0(\hat{w}-\varepsilon)}{F^1(\hat{w}) - F^1(\hat{w}-\varepsilon)} \frac{1-\eta}{\eta}\right)^{-1}. \tag{10}$$

Suppose only G_U is discontinuous at $\hat{w}$. That is, $\lim_{\varepsilon\to 0} G_U(\hat{w}) - G_U(\hat{w}-\varepsilon) > 0$ and $\lim_{\varepsilon\to 0} G_I(\hat{w}) - G_I(\hat{w}-\varepsilon) = 0$. Then $q(\hat{w}) = p(\hat{w})$. Similarly, if only G_I is discontinuous at $\hat{w}$, $q(\hat{w}) = 0$. If both are discontinuous, then $q(\hat{w}) > 0$ and its value depends on the ratio of the size of the jump in G_U and G_I.

If $q(\hat{w}) = 0$, it must be $\lim_{\varepsilon\to 0} \left(F^0(\hat{w}) - F^0(\hat{w}-\varepsilon)\right) / \left(F^1(\hat{w}) - F^1(\hat{w}-\varepsilon)\right) = \infty$, which holds only if G_I is discontinuous at $\hat{w}$ and G_U is continuous. If $q(\hat{w}) = p(\hat{w})$, G_I is continuous.

A.1.3 (II) Necessary conditions

Proposition 5. *In any equilibrium, there exists $0 \leq \overline{w} < \infty$ such that the uninformed agent stops with probability 1 at $\overline{w}$ and the informed randomizes over $[0, \overline{w}]$.*

Proof of Proposition 5. First, I show in Lemma 6 that if the uninformed agent were ever to stop at some finite time, he stops with probability 1 at that time. Given the uninformed agent's behavior, I then show that the informed agent randomizes before the uninformed agent stops and stops with probability 1 by the time the uniformed agent stops with probability 1 (Lemma 7 and Lemma 8). Finally, the informed agent's equilibrium behavior in turn implies the uninformed agent must stop in finite time (Lemma 11).

Denote by $\overline{w}_0 = \inf\{w : G_U(w) > 0\}$, the first time the uninformed agent stops. The following result shows if the uninformed agent were to stop at some finite time, he stops with probability 1 at that time. Intuitively, at the time the uninformed agent stops, if the informed agent does not stop or stop continuously, then the uninformed agent stopping with positive probability reveals that he is uninformed, which dominates stopping at any time afterwards. If the informed agent stops with positive probability, then the uninformed agent would rather stop with positive probability at $\overline{w}_0 + \varepsilon$ instead of pooling with the informed at $\overline{w}_0$.

Lemma 6. *In any equilibrium, if $\overline{w}_0 < \infty$, then $G_U(\overline{w}_0) = 1$.*

Proof. Suppose $G_I(w)$ is continuous at $\overline{w}_0$. Suppose the uninformed agent stops at $\overline{w}_0$ with positive probability, that is, if $G_U(w)$ is discontinuous at $\overline{w}_0$. By (10), $q(\overline{w}_0) = p(\overline{w}_0)$. Because $U(\cdot)$ is increasing in $q(\cdot)$ and $q(w) \leq p(w)$ for all $w \geq 0$, the uniformed agent's expected payoff from stopping at any $w \geq \overline{w}_0$ is bounded above by the expected payoff evaluated at $q(w) = p(w)$ for all $w \geq \overline{w}_0$, denoted by $\overline{U}(w)$. The derivative of $\overline{U}(w)$ with respect to w is given by

$$\overline{U}'(w) = e^{-(\beta+r+\lambda)w}\left(-(1+\rho)r\kappa - e^{\lambda w}(1+\kappa)r\rho\right) < 0.$$

This means that if the uninformed agent stops with positive probability at $\overline{w}_0$, his expected payoff from stopping at $\overline{w}_0$ is strictly higher than upper bound of his expected payoff from stopping at any $w > \overline{w}_0$. Therefore, if $G_I(w)$ is continuous at $\overline{w}_0$, the uninformed agent's best response is to stop with probability 1 at $\overline{w}_0$, that is, $G_U(\overline{w}_0) = 1$.

Suppose $G_I(w)$ is discontinuous at $\overline{w}_0$. Suppose the contrary $G_U(\overline{w}_0) < 1$. Then there exists $\tilde{w} > \overline{w}_0$ (where $\tilde{w}$ can be infinity) such that $G_U(\tilde{w}) = 1$. That is to say, stopping at any $\overline{w}_0 < w \leq \tilde{w}$ is on the equilibrium path.

Because the uninformed agent stops at $\overline{w}_0$, so either $G_U(w)$ is continuous at $\overline{w}_0$, in which case $q(\overline{w}_0) = 0$, or $G_U(w)$ is discontinuous at $\overline{w}_0$, where $q(\overline{w}_0) < p(\overline{w}_0)$. By definition, $G_I(w)$ is right-continuous, which means $\lim_{\varepsilon\to 0} G_I(\overline{w}_0+\varepsilon) = G_I(\overline{w}_0)$. Suppose the uninformed agent stops at $\overline{w}_0+\varepsilon$ with positive probability, that is, $G_U(w)$ is discontinuous at $\overline{w}_0+\varepsilon$. Because stopping at $\overline{w}_0+\varepsilon$ is on the equilibrium path, $\lim_{\varepsilon\to 0} q(\overline{w}_0+\varepsilon) = \lim_{\varepsilon\to 0} p(\overline{w}_0+\varepsilon) = p(\overline{w}_0) > q(\overline{w}_0)$.[19] This implies $\lim_{\varepsilon\to 0} U(\overline{w}_0+\varepsilon) = \kappa + \lim_{\varepsilon\to 0} q(\overline{w}_0+\varepsilon) > \kappa + q(\overline{w}_0) = U(\overline{w}_0)$. □

Thus, if $G_I(w)$ is discontinuous at $\overline{w}_0$, then stopping at $\overline{w}_0$ is strictly dominated by stopping with positive probability at $\overline{w}_0+\varepsilon$, which contradicts the definition of $\overline{w}_0$.

Following Lemma 6, define $\overline{w}$ as the time $G_U(w)$ jumps up to 1. That is,

$$\overline{w} := \inf\left\{w : G_U(w) = 1\right\}.$$

Note that $\overline{w} \in [0,\infty]$. Lemma 6 says that either the uninformed agent never stops ($\overline{w} = \infty$), or the uninformed agent does not stop at any $w \in [0,\overline{w})$ and stops with probability 1 at $\overline{w}$. Then (8) and (9) become

$$F^1(w) = \begin{cases} 1 - e^{-\beta w} & w \in [0,\overline{w}) \\ 1 & w = \overline{w} \end{cases} \text{ and } F^0(w) = \begin{cases} 1 - e^{-\beta w}\left(1-\gamma G_I(w)\right) & w \in [0,\overline{w}) \\ 1 & w = \overline{w} \end{cases}. \quad (11)$$

[19] To be specific, because stopping at $\overline{w}_0+\varepsilon$ is on the equilibrium path, for the uninformed agent, stopping with positive probability at a time the informed agent doesn't reveals that the agent is uninformed, that is, $\lim_{\varepsilon\to 0} q(\overline{w}_0+\varepsilon) = \lim_{\varepsilon\to 0} p(\overline{w}_0+\varepsilon)$. If stopping at $\overline{w}_0+\varepsilon$ is off the equilibrium path, the decision maker's belief $q(\overline{w}_0+\varepsilon)$ does not necessarily need to be $p(\overline{w}_0+\varepsilon)$.

The following result shows that if the uninformed agent were to stop in finite time ($\overline{w} < \infty$), the informed agent does not stop at $w > \overline{w}$ as doing so reveals himself to be the informed.

Lemma 7. *In any equilibrium, if $\overline{w} < \infty$, then $G_I(w) = 1$ for all $w \geq \overline{w}$.*

Proof. By the definition of $\overline{w}$, $G_U(w) = 1$ for all $w \geq \overline{w}$. If the informed agent stops at any any $w > \overline{w}$, then the decision maker's belief about the agent if disclosure stops at $w > \overline{w}$ is that the agent is informed—because the uninformed agent has stopped with probability 1 by time $\overline{w}$. From the perspective of time $\overline{w}$, if the informed agent stops at $\overline{w}$, his expected payoff is $\kappa + q(\overline{w})$, where $q(\overline{w}) > 0$. If the informed agent stops at any $w > \overline{w}$, his expected payoff is

$$\kappa\left(\int_{\overline{w}}^{w} \lambda e^{-(r+\lambda+\beta)(s-\overline{w})}\mathrm{d}s + \int_{\overline{w}}^{w} \beta e^{-(r+\lambda+\beta)(s-\overline{w})}\mathrm{d}s + e^{-(r+\lambda+\beta)(w-\overline{w})}\right) < \kappa < \kappa + q(\overline{w}).$$

Therefore, stopping at any $w > \overline{w}$ is dominated by stopping at $\overline{w}$. This implies that the informed agent stops with probability 1 by $\overline{w}$, that is, $G_I(\overline{w}) = 1$. □

Lemma 7 establishes the informed agent's behavior at and after $\overline{w}$. The following result establishes the informed agent's behavior before the uninformed agent stops—the informed agent is indifferent . Intuitively, given that the uninformed agent does not stop in $[0, \overline{w})$, if the informed agent stops with a strictly positive probability in $[0, \overline{w})$, it reveals that the agent is informed. So the informed agent must stop continuously. If there exists an interval of time during which the informed agent does not stop, then stopping in this interval with positive probability reveals the agent is uninformed. This means the uninformed agent would deviate to stopping in this interval. The following lemma formalize this intuition.

Lemma 8. *In any equilibrium, $G_I(w)$ is continuous and strictly increasing in w for $w \in [0, \overline{w})$.*

Proof. Suppose there $\hat{w} \in [0, \overline{w})$ such that $G_I(w)$ is discontinuous. That is, $\lim_{\varepsilon\to 0} G_I(\hat{w}-\varepsilon) < G_I(\hat{w})$.[20] By (10), $q(\hat{w}) = 0$. So from the perspective of $\hat{w}$, the informed agent's expected payoff from stopping at $\hat{w}$ is κ. If the informed agent stops at $\hat{w} + \varepsilon$ for $\varepsilon > 0$ small, the decision maker's belief is $q(\hat{w} + \varepsilon) > 0$. The informed agent's expected payoff is then $\kappa + \lim_{\varepsilon\to 0} q(\hat{w} + \varepsilon) > \kappa$. Therefore, stopping at $\hat{w}$ is dominated by stopping at $\hat{w} + \varepsilon$, which contradicts the premise that the informed agent stops at $\hat{w}$ with positive probability.

Suppose there exists $[w_1, w_2]$ with $0 \leq w_1 < w_2 < \overline{w}$ such that $G_I(w)$ is constant for $w \in [w_1, w_2]$. In words, the informed agent does not stop at any $w \in [w_1, w_2]$. Let $\tilde{w}$ be the supremum of w_2 for which over $[w_1, w_2]$, $G_I(w)$ is constant. There are two cases. Either $G_I(w)$ is constant with a value strictly less than 1, or $G_I(w)$ is constant with value 1.

[20]In the case of $\hat{w} = 0$, $G_I(\hat{w}) = G_I(0) > 0$.

First, consider the case where $G_I(w) < 1$ for $w \in [w_1, \tilde{w}]$. By Lemma 6, the uninformed agent does not stop in $[w_1, \tilde{w}]$ either. So if disclosure stops in $[w_1, \tilde{w}]$, it can only be due to exogenous termination, which is uninformative of the state. The decision maker's belief that $\theta = 1$ if disclosure stops at any $w \in [w_1, \tilde{w}]$ is

$$q(w) = \frac{\eta}{\eta + e^{-\lambda w}\left(1 - \gamma G_I(w_1)\right)\right)(1-\eta)},$$

which means $q'(w) = \lambda(1 - q(w))q(w)$. Given $q(w)$ is differentiable in w for $w \in (w_1, \tilde{w})$, the informed agent's expected payoff from stopping at $w \in [w_1, \tilde{w}]$ is differentiable for $w \in (w_1, \tilde{w})$ with derivative proportional to

$$q'(w) - r\kappa - (r + \lambda)\, q(w) < 0,$$

which means $V(\tilde{w}) < V(w_1)$. By part (i), $G_I(w)$ is continuous in w which means $V(w)$ is continuous in w for a neighborhood around $\tilde{w}$, $w \in (\tilde{w}, \tilde{w} + \varepsilon)$. So $\lim_{\varepsilon \to 0} V(\tilde{w} + \varepsilon) = V(\tilde{w}) < V(w_1)$. In words, stopping at any $w \in (\tilde{w}, \tilde{w} + \varepsilon)$ is dominated by stopping at w_1, which means $G_I(w)$ is constant over the interval $[\tilde{w}, \tilde{w} + \varepsilon]$. This contradicts the definition of $\tilde{w}$.

Next, consider the case where $G_I(w) = 1$ for $w \in [w_1, \tilde{w}]$. Then $G_I(w) = 1$ for all $w \geq w_1$. Then for the uninformed agent, stopping with positive probability at any $w > w_1$ reveals that the agent is uninformed. By the same argument as in Lemma 6, stopping with probability 1 at $\overline{w}$ is dominated by stopping with positive probability at any $w \in (w_1, \tilde{w}]$ where $\tilde{w} < \overline{w}$. This contradicts the definition of $\overline{w}$. □

It follows from Lemma 8 that the informed agent must stop continuously in $[0, \overline{w})$ and this is true regardless of whether $\overline{w}$ is finite or not. In other words, the informed agent must be indifferent with respect to stopping at any $w \in [0, \overline{w}]$. It follows from the following lemma that while the informed agent is indifferent between stopping at w and $w + \varepsilon$, the uninformed agent strictly prefers stopping at $w + \varepsilon$.

To simplify notation, define

$$\Delta V(w) \coloneqq \lim_{\varepsilon \to 0} \frac{V(w + \varepsilon) - V(w)}{\varepsilon} \text{ and } \Delta U(w) \coloneqq \lim_{\varepsilon \to 0} \frac{U(w + \varepsilon) - U(w)}{\varepsilon}.$$

Lemma 9. *Fix any $w \geq 0$ and $\varepsilon > 0$. If $\Delta V(w) \geq 0$, then $\Delta U(w) \geq 0$. Moreover, if $\lim_{\varepsilon \to 0} q(w + \varepsilon) > 0$, $\Delta V(w) \geq 0$ implies $\Delta U(w) > 0$.*

Proof. From the perspective of w, if the agent (informed and uninformed) stops at w, he gets payoff $\kappa + q(w)$. If informed agent stops at $w + \varepsilon$, his expected payoff is

$$\left(1 - e^{-\lambda\varepsilon}\right) e^{-\beta\varepsilon}\kappa + e^{-\lambda\varepsilon}\left(1 - e^{-\beta\varepsilon}\right)(\kappa + q(w + \varepsilon)) + e^{-\lambda\varepsilon} e^{-\beta\varepsilon} e^{-r\varepsilon}(\kappa + q(w + \varepsilon)).$$

Using a Taylor expansion $e^{-r\varepsilon} = 1 - r\varepsilon$ and ignoring terms with orders ε^2 and higher,

$$\Delta V(w) = -r\kappa - \lim_{\varepsilon \to 0} q(w+\varepsilon)(r+\lambda) + \lim_{\varepsilon \to 0} \frac{q(w+\varepsilon) - q(w)}{\varepsilon}.$$

If uninformed agent stops at $w + \varepsilon$, his expected payoff is

$$\begin{aligned}
&(1-p(w))\left(1-e^{-\lambda\varepsilon}\right)e^{-\beta\varepsilon}\kappa \\
&+\left((1-p(w))\,e^{-\lambda\varepsilon}+p(w)\right)\left(1-e^{-\beta\varepsilon}\right)(\kappa+q(w+\varepsilon)) \\
&+\left((1-p(w))\,e^{-\lambda\varepsilon}+p(w)\right)e^{-\beta\varepsilon}e^{-r\varepsilon}(\kappa+q(w+\varepsilon))\,.
\end{aligned}$$

Then

$$\Delta U(w) = -r\kappa - (r+\lambda)\lim_{\varepsilon \to 0} q(w+\varepsilon) + \lambda p(w) \lim_{\varepsilon \to 0} q(w+\varepsilon) + \lim_{\varepsilon \to 0} \frac{q(w+\varepsilon) - q(w)}{\varepsilon}.$$

So if $\Delta V(w) \geq 0$, then $\Delta U(w) \geq 0$. Moreover, if $\lim_{\varepsilon \to 0} q(w+\varepsilon) > 0$, $\Delta U(w) > 0$.

Also, $\Delta V(w) \geq 0$ implies

$$\lim_{\varepsilon \to 0} \frac{q(w+\varepsilon) - q(w)}{\varepsilon} \geq r\kappa + \lim_{\varepsilon \to 0} q(w+\varepsilon)(r+\lambda) > 0,$$

which implies $\lim_{\varepsilon \to 0} q(w+\varepsilon) > q(w)$. Therefore, if $q(w) > 0$, $\lim_{\varepsilon \to 0} q(w+\varepsilon) \geq q(w) > 0$. □

Given that the informed agent is indifferent and that $G_I(w)$ is continuous in w, $q(w) > 0$. So $\Delta V(w) = 0$ implies $\Delta U(w) > 0$.

Lemma 10. *In any equilibrium, $G_I(w)$ is twice differentiable in w for all $w \in (0, \overline{w})$.*

Proof. First, by the Lebesgue's theorem (see, for example, Royden and Fitzpatrick, 1988), because $G_I(w)$ is continuous and monotone so it is almost everywhere differentiable in w for $w \in (0, \overline{w})$. This means $q(w)$ is continuous and almost everywhere differentiable in w. Note that $V(\cdot)$ is continuous in w and $q(\cdot)$. It follows from Lemma 8 that the informed agent is indifferent in $w \in [0, \overline{w}]$ which implies $V(\cdot)$ is constant in w for $w \in [0, \overline{w}]$. That is, $\lim_{\varepsilon \to 0} V(w+\varepsilon) = \lim_{\varepsilon \to 0} V(w-\varepsilon) = V(w)$ for all w. This implies

$$\lim_{\varepsilon \to 0} \frac{q(w+\varepsilon) - q(w)}{\varepsilon} = \lim_{\varepsilon \to 0} \frac{q(w) - q(w-\varepsilon)}{\varepsilon} = r\kappa + (r+\lambda)q(w),$$

which means $q(w)$ is everywhere differentiable in w.

$$q'(w) = r\kappa + (r+\lambda)q(w). \tag{12}$$

By (11), $F^0(\cdot)$ is a differentiable function of $G_I(w)$ and w and is therefore almost everywhere differentiable in w. Denote the derivative of $F^0(w)$, whenever exists, by $f^0(w)$, and the derivative of $F^1(w)$ is $f^1(w) = \beta e^{-\beta w}$. Then the decision maker's belief that $\theta = 1$ is

$$q(w) = \frac{\beta e^{-\beta w}\eta}{\beta e^{-\beta w}\eta + e^{-\lambda w} f^0(w)(1-\eta)}, \tag{13}$$

where $f^0(w) = (1-\gamma)\beta e^{-\beta w} + \gamma\left[G_I'(w)e^{-\beta w} + \beta e^{-\beta w}(1 - G_I(w))\right]$. Because $q(w)$ is everywhere differentiable in w, $f^0(w)$ and therefore $G_I'(w)$ is everywhere differentiable in w.

Plugging (13) into (12), one obtains a second-order differential equation in $G_I(w)$, $G_I''(w) = \mathscr{G}(G_I(w), G_I'(w), w)$, where $\mathscr{G}$ is given by

$$\begin{aligned}
&\mathscr{G}(G_I(w), G_I'(w), w) \qquad (14)\\
=&\beta G_I'(w) - \frac{\beta\rho}{\rho-\eta}\left(r(1-\eta) + e^{\lambda w}\eta\left(r\kappa\left(\frac{\eta + e^{-\lambda w}(1-\eta)}{\eta}\right)^2 + r + \lambda\right)\right)\\
&+ r\left(1 + 2\kappa\left(\frac{\eta + e^{-\lambda w}(1-\eta)}{\eta}\right)\right)\left(\beta G_I(w) - G_I'(w)\right)\\
&- \frac{r\kappa e^{-\lambda w}(\rho-\eta)}{\beta\rho\eta}\left(\beta G_I(w) - G_I'(w)\right)^2.
\end{aligned}$$

Recall that given that disclosure starts at t and given the agent's equilibrium starting strategies, $\rho = \rho(t)$ and $\eta = \mu$, which is the same $\mathscr{G}$ given in footnote 10. □

The following result establishes that the informed agent cannot be indifferent for all $w \geq 0$. In other words, disclosure must stop in finite time with probability 1.

Lemma 11. *In any equilibrium, $\overline{w} < \infty$.*

Proof. Recall that by Lemma 10, in equilibrium, $q'(w) = r\kappa + (r+\lambda)q(w)$ for $w \in (0, \overline{w})$. This means $q(w)$ must increase exponentially in w. In equilibrium, $q(w)$ is bounded above by $p(w) < 1$ for all $w \geq 0$. In other words, there does not exist $q(w)$ such that $q'(w) = r\kappa + (r+\lambda)q(w)$ for all $w \geq 0$. This implies $\overline{w} < \infty$. □

Divinity. By Proposition 5, both types of agent stop by $\overline{w} < \infty$. This means that stopping at $w > \overline{w}$ is off the equilibrium path. First, I show that if an equilibrium survives the divinity refinement, the decision maker's belief that $\theta = 1$ if disclosure stops off-path at $w > \overline{w}$ is equal to the uninformed agent's belief that $\theta = 1$, $p(w)$. Next, I show that the informed agent's probability of stopping must be continuous for all $w \in [0, \overline{w}]$.

Lemma 12. *In a divine equilibrium, the decision maker's belief that $\theta = 1$ if disclosure stops at $w > \overline{w}$ is $q(w) = p(w)$ for $\overline{w} \geq 0$.*

Proof. From the perspective of time $\overline{w}$, the agent's expected payoff is given by $V(w)$ and $U(w)$ where both the uninformed agent's and the decision maker's belief that $\theta = 1$ at $\overline{w}$ is given by $p(\overline{w})$. It follows from Lemma 9 that for any decision maker's belief $q(w)$ for $w > \overline{w}$, if $V(w)$ increases in w, then $U(w)$ increases in w. That is, if the informed agent finds it optimal to deviate from stopping at $\overline{w}$ to stopping at $w > \overline{w}$, the uninformed agent also finds it optimal to deviate. As mentioned in the paper, the divinity refinement prescribes that the decision maker's off-path belief should assign zero weight to the type of agent that has less incentive to deviate. The result follows. □

Lemma 13. *In a divine equilibrium, for any $\overline{w} \geq 0$, $q(\overline{w}) = p(\overline{w})$. This means $G_I(w)$ is continuous at $w = \overline{w}$.*

Proof. By Lemma 12, from the perspective of time $\overline{w}$, if the uninformed agent stops at $\overline{w}$, his expected payoff is $\kappa + q(\overline{w})$. By Lemma 12, $\lim_{\varepsilon\to 0} q(\overline{w}+\varepsilon) = p(\overline{w})$, and if the uninformed agent stops at $\overline{w}+\varepsilon$ for $\varepsilon > 0$ small, his expected payoff is

$$\lim_{\varepsilon\to 0} (1-p(\overline{w}))\left(1-e^{-\lambda\varepsilon}\right)\kappa + \left((1-p(\overline{w}))\,e^{-\lambda\varepsilon} + p(\overline{w})\right)e^{-r\varepsilon}\left(\kappa + q(\overline{w}+\varepsilon)\right) = \kappa + p(\overline{w}).$$

This implies for any $\overline{w} \geq 0$, $\kappa + q(\overline{w}) = \kappa + p(\overline{w})$: because if $q(\overline{w}) < p(\overline{w})$, the uninformed agent finds it profitable to deviate to stopping at $\overline{w}+\varepsilon$, violating the definition of $\overline{w}$.

For $\overline{w} = 0$, $G_I(w) = 1$ for all $w \geq 0$ and is thus continuous at $\overline{w} = 0$ by definition. For $\overline{w} > 0$, at $\overline{w}$, $\Pr(w_{\text{stop}} = \overline{w}|\theta = 1) = 1$ and $\Pr(w_{\text{stop}} = \overline{w}|\theta = 0) = (1-\gamma)\cdot 1 + \gamma(G_I(\overline{w}) - G_I(\overline{w}^-))$, where $G_I(\overline{w}^-) = \lim_{\varepsilon\to 0} G_I(\overline{w}-\varepsilon)$ is the left limit of G_I at $\overline{w}$. Recall that γ is the probability that the agent is informed at the beginning of the continuation game conditional on $\theta = 0$, and is given by $\eta = \rho(1-\gamma(1-\eta))$. Therefore,

$$q(\overline{w}) = \frac{\eta}{\eta + e^{-\lambda\overline{w}}\left((1-\gamma) + \gamma(G_I(\overline{w}) - G_I(\overline{w}^-))\right)(1-\eta)}.$$

Thus, $q(\overline{w}) = p(\overline{w})$ if and only if $G_I(\overline{w}) = G_I(\overline{w}^-)$. □

Lemma 14. *In any divine equilibrium, $\overline{w} = 0$ if and only if $\rho = \eta$. So for any $\rho > \eta$, $\overline{w} > 0$.*

Proof. By Lemma 12, $q(\overline{w}) = p(\overline{w})$. Suppose $\overline{w} = 0$, then $G_I(0) = 1$ and $G_U(0) = 1$ for all $w \geq 0$. Then $F^1(0) = F^0(0) = 1$ (recall that $F^\theta(w)$ is the probability that disclosure stops by w conditional on state θ). This means

$$q(\overline{w}) = q(0) = \frac{(F^1(0)-0)\eta}{(F^1(0)-0)\eta + e^{-\lambda\cdot 0}(F^0(0)-0)(1-\eta)} = \eta.$$

On the other hand, $p(\overline{w}) = \rho(0) = \rho$. So for $\overline{w} = 0$, $q(\overline{w}) = p(\overline{w})$ if and only if $\rho = \eta$. Therefore, for any $\rho > \eta$, $\overline{w} > 0$. □

Boundary conditions. By Lemma 8 (i), $G_I(0) = 0$. By Proposition 5, $G_I(\overline{w}) = 1$. By Lemma 13, $G_I(w)$ is continuous for all $w \in [0, \overline{w}]$ and $q(\overline{w}) = p(\overline{w})$, which implies $\lim_{\varepsilon \to 0} q(\overline{w} - \varepsilon) = q(\overline{w}) = p(\overline{w})$. Writing out $p(\overline{w}) = q(\overline{w})$,

$$\frac{\rho}{\rho + e^{-\lambda \overline{w}}(1-\rho)} = \frac{\beta\eta}{\beta\eta + e^{-\lambda\overline{w}}\left((1-\gamma)\beta + \gamma\left(G_I'(\overline{w}) + \beta(1 - G_I(\overline{w}))\right)\right)(1-\eta)}.$$

Because $G_I(\overline{w}) = 1$, the above equality implies $G_I'(\overline{w}) = 0$.

To sum up, the boundary conditions are given by $G_I(0) = 0$, $G_I(\overline{w}) = 1$, and $G_I'(\overline{w}) = 0$.

Therefore, in any equilibrium, for all $w \in [0, \overline{w}]$

$$f^1(w) = \beta e^{-\beta w} \text{ and } f^0(w) = (1-\gamma)\beta e^{-\beta w} + \gamma\left(G_I'(w)e^{-\beta w} + \beta e^{-\beta w}(1 - G_I(w))\right).$$

Substituting these expressions for $f^1(w)$ and $f^0(w)$ into (13), the decision maker's belief that $\theta = 1$ when disclosure stops at $w \in [0, \overline{w}]$ can be written as

$$q(w) = \frac{\beta\eta}{\beta\eta + e^{-\lambda w}\left((1-\gamma)\beta + \gamma\left(G_I'(w) + \beta(1 - G_I(w))\right)\right)(1-\eta)}. \tag{15}$$

A.1.3 (III) Existence and uniqueness

The following theorem establishes existence and uniqueness of a solution to the desired boundary value problem. Moreover, this unique solution is an equilibrium.

Theorem 2. *There exists a unique solution, w^* and $G_I^*(w)$, to the following boundary value problem: for all $w \in [0, w^*]$,*

$$G_I''(w) = \mathscr{G}(G_I(w), G_I'(w), w), \quad G_I(0) = 0, G_I(w^*) = 1, \text{ and } G_I'(w^*) = 0, \tag{BVP}$$

where $\mathscr{G}$ is given by (14).

To prove this, I first present a useful lemma that establishes equivalence between two boundary value problems (or initial value problems).

Lemma 15. *There exists $0 < \overline{w}_{\max} < \infty$ such that for any $\overline{w} \in (0, \overline{w}_{\max})$, there exists a unique solution to the initial value problem (BVP-q): for all $w \in [0, \overline{w}]$,*

$$q'(w) = r\kappa + (r + \lambda)q(w), \quad q(\overline{w}) = p(\overline{w}) \tag{BVP-q}$$

and a unique solution to the initial value problem (BVP-1): for all $w \in [0, \overline{w}]$,

$$G_I''(w) = \mathscr{G}(G_I(w), G_I'(w), w), \quad G_I(\overline{w}) = 1 \text{ and } G_I'(\overline{w}) = 0. \tag{BVP-1}$$

Moreover, given the solution to (BVP-1), the corresponding $q(w)$ given by (15) solves (BVP-q) and vice versa.

Proof. First, I show these two initial value problems are equivalent.

Equation (15) links $G_I(w)$ with $q(w)$. In equilibrium, the informed agent's indifference condition $V'(w) = 0$ reduces to $q'(w) = r\kappa + (r+\lambda)q(w)$, which is the differential equation in (BVP-q). This differential equation is equivalent to the differential equation in (BVP-1). Moreover, the boundary conditions $G_I(w^*) = 1$ and $G_I'(w^*) = 0$ imply $q(w^*) = p(w^*)$.

Next, I derive conditions under which each initial value problem admits a unique solution.

By the Picard-Lindelöf Theorem (see, for example, Teschl, 2012, Theorem 2.2), for any $\overline{w} > 0$, (BVP-q) has a unique solution. In fact, the solution admits a closed form. To emphasize this solution depends on $\overline{w}$, denote it by $\overline{q}(w;\overline{w})$, where

$$\overline{q}(w;\overline{w}) = \left(p(\overline{w}) + \frac{r\kappa}{r+\lambda}\right)e^{-(r+\lambda)(\overline{w}-w)} - \frac{r\kappa}{r+\lambda}. \tag{16}$$

For (BVP-1), because $G_I(w)$ is twice differentiable for $w \in (0,\overline{w})$, $G'(w) < \infty$ and $G(w) < \infty$ for all $w \in (0,\overline{w})$. By (15), this means there does not exists $w \in (0,\overline{w})$ such that $q(w) = 0$. If (BVP-1) admits a solution, $\overline{w}$ must be $q(w) > 0$ for all $w \in [0,\overline{w}]$. This means $\overline{q}(w;\overline{w}) > 0$ for all $w \in [0,\overline{w}]$. I now derive conditions under which $\overline{q}(w;\overline{w}) > 0$ for all $w \in [0,\overline{w}]$.

By (16), $\overline{q}(w;\overline{w})$ is strictly increasing in w and $\overline{q}(\overline{w};\overline{w}) = p(\overline{w}) > 0$, so $q(w;\overline{w}) > 0$ for all w as long as $q(0;\overline{w}) > 0$.

Claim 1. There exists a unique $\overline{w}_{\max} > 0$ such that $\overline{q}(0;\overline{w}) > 0$ if and only if $\overline{w} < \overline{w}_{\max}$.

Proof of the claim. Plugging in $w = 0$ into (16),

$$\overline{q}(0;\overline{w}) = \left(p(\overline{w}) + \frac{r\kappa}{r+\lambda}\right)e^{-(r+\lambda)\overline{w}} - \frac{r\kappa}{r+\lambda}.$$

Then

$$\lim_{\overline{w}\to 0}\overline{q}(0;\overline{w}) = \overline{q}(0;0) = \rho > 0 \text{ and } \lim_{\overline{w}\to\infty}\overline{q}(0;\overline{w}) = -\frac{r\kappa}{r+\lambda} < 0.$$

It can be verified that the derivative of $\overline{q}(0;\overline{w})$ with respect to $\overline{w}$ is strictly negative. Because $\overline{q}(0;\overline{w})$ is continuous in $\overline{w}$, by the intermediate value theorem, there exists a unique $\overline{w}_{\max}$ such that $\overline{q}(0;\overline{w}_{\max}) = 0$, $\overline{q}(0;\overline{w}) > 0$ for all $\overline{w} < \overline{w}_{\max}$, and $\overline{q}(0;\overline{w}) < 0$ for all $\overline{w} > \overline{w}_{\max}$. □

By the Picard-Lindelöf Theorem, (BVP-1) has a unique solution for $\overline{w} < \overline{w}_{\max}$. □

Lemma 15 is useful because (BVP-q) has a closed-form solution. Exploiting the relationship between $G_I(w)$ and $q(w)$, one obtains properties of $G_I(w)$ that are otherwise hard to derive.

Lemma 16. *Let $\overline{G}_I(w;\overline{w})$ be the (unique) solution to (BVP-1) for some $\overline{w} \in (0, \overline{w}_{\max})$. There exists a unique $w^* \in (0, \overline{w}_{\max})$ such that $\overline{G}_I(0; w^*) = 0$.*

Proof. By Lemma 15, I derive properties of the (unique) solution to (BVP-1) through (16) via (15). Equate (16) with (15),

$$G'_I(w;\overline{w}) = \beta G_I(w;\overline{w}) - \frac{\beta\rho}{\rho-\eta}\left(1-\eta+e^{\lambda w}\eta\left(1-\frac{1}{\overline{q}(w;\overline{w})}\right)\right). \tag{17}$$

Note that this condition is in itself a differential equation in $G_I(w;\overline{w})$ and the solution to this differential equation with $G_I(\overline{w};\overline{w}) = 1$ is the solution to (BVP-1) for a fixed $\overline{w}$. More specifically, the initial value problem, (BVP-2),

$$G'_I(w;\overline{w}) = \beta G_I(w;\overline{w}) - \frac{\beta\rho}{\rho-\eta}\left(1-\eta+e^{\lambda w}\eta\left(1-\frac{1}{\overline{q}(w;\overline{w})}\right)\right),\ G_I(\overline{w};\overline{w}) = 1 \tag{BVP-2}$$

is equivalent to (BVP-q) and thus is equivalent to (BVP-1). Define

$$H(w;\overline{w}) := \frac{\beta\rho}{\rho-\eta}\left(1-\eta+e^{\lambda w}\eta\left(1-\frac{1}{\overline{q}(w;\overline{w})}\right)\right). \tag{18}$$

Then (17) becomes $G'_I(w;\overline{w}) = \beta G_I(w;\overline{w}) - H(w;\overline{w})$, which has a closed-form solution, denoted by

$$\overline{G}_I(w;\overline{w}) = ce^{\beta w} - e^{\beta w}\int_0^w e^{-\beta s}H(s;\overline{w})\mathrm{d}s,$$

where $c \in \mathbb{R}$ is an integration constant to be determined. By $G_I(\overline{w};\overline{w}) = 1$,

$$1 = ce^{\beta\overline{w}} - e^{\beta\overline{w}}\int_0^{\overline{w}} e^{-\beta s}H(s;\overline{w})\mathrm{d}s \implies c = e^{-\beta\overline{w}} + \int_0^{\overline{w}} e^{-\beta s}H(s;\overline{w})\mathrm{d}s.$$

Therefore,

$$\overline{G}_I(w;\overline{w}) = \left(e^{-\beta\overline{w}} + \int_0^{\overline{w}} e^{-\beta s}H(s;\overline{w})\mathrm{d}s\right)e^{\beta w} - e^{\beta w}\int_0^w e^{-\beta s}H(s;\overline{w})\mathrm{d}s. \tag{19}$$

By definition, for all $\overline{w} \in (0, \overline{w}_{\max})$ and all $w < \overline{w}$, $H(w;\overline{w}) < \infty$. Set $w = 0$,

$$\overline{G}_I(0;\overline{w}) = e^{-\beta\overline{w}} + \int_0^{\overline{w}} e^{-\beta s}H(s;\overline{w})\mathrm{d}s.$$

The goal is to show there exists a unique $w^* \in (0, \overline{w}_{\max})$ such that $\overline{G}_I(0; w^*) = 0$. The proof uses the intermediate value theorem.

Take the derivative $\overline{G}_I(0;\overline{w})$ with respect to $\overline{w}$,

$$\frac{\mathrm{d}}{\mathrm{d}\overline{w}}\overline{G}_I(0;\overline{w}) = -\beta e^{-\beta\overline{w}} + e^{-\beta\overline{w}}H(\overline{w};\overline{w}) + \int_0^{\overline{w}} e^{-\beta s}\frac{\partial}{\partial\overline{w}}H(s;\overline{w})\mathrm{d}s < 0. \tag{20}$$

This derivative is negative because $H(\overline{w};\overline{w}) = \beta$; and because $\partial\overline{q}(s;\overline{w})/\partial\overline{w} < 0$ for $s \leq \overline{w}$, $\partial H(s;\overline{w})/\partial\overline{w} < 0$. Let $\overline{w} \downarrow 0$. Then

$$\lim_{\overline{w}\downarrow 0}\overline{G}_I(0;\overline{w}) = \lim_{\overline{w}\downarrow 0} e^{-\beta\overline{w}} + \lim_{\overline{w}\downarrow 0}\int_0^{\overline{w}} e^{-\beta s}H(s;\overline{w})\mathrm{d}s = 1.$$

Let $\overline{w} \uparrow \overline{w}_{\max}$. Then

$$\lim_{\overline{w}\uparrow\overline{w}_{\max}}\overline{G}_I(0;\overline{w}) = e^{-\beta\overline{w}_{\max}} + \lim_{\overline{w}\uparrow\overline{w}_{\max}}\int_0^{\overline{w}} e^{-\beta s}H(s;\overline{w})\mathrm{d}s.$$

Claim 2. The following statement is true:

$$\lim_{\overline{w}\uparrow\overline{w}_{\max}}\int_0^{\overline{w}} e^{-\beta s}H(s;\overline{w})\mathrm{d}s = -\infty.$$

Proof of the claim. Substituting in the definition of $H(s;\overline{w})$, given by (18),

$$\lim_{\overline{w}\uparrow\overline{w}_{\max}}\int_0^{\overline{w}} e^{-\beta s}H(s;\overline{w})\mathrm{d}s = \lim_{\overline{w}\uparrow\overline{w}_{\max}}\int_0^{\overline{w}} e^{-\beta s}\frac{\beta\rho}{\rho-\eta}\left(1-\eta+e^{\lambda s}\eta\right)\mathrm{d}s - \lim_{\overline{w}\uparrow\overline{w}_{\max}}\frac{\beta\rho\eta}{\rho-\eta}\int_0^{\overline{w}}\frac{e^{-\beta s}e^{\lambda s}}{\overline{q}(s;\overline{w})}\mathrm{d}s.$$

The first limit is finite. It suffices to show the second limit is infinite. Because $\overline{q}(s;\overline{w}) > 0$ for all $s \in (0,\overline{w})$,

$$\lim_{\overline{w}\uparrow\overline{w}_{\max}}\int_0^{\overline{w}}\frac{e^{-\beta s}e^{\lambda s}}{\overline{q}(s;\overline{w})}\mathrm{d}s \geq \lim_{\overline{w}\uparrow\overline{w}_{\max}}\int_0^{\overline{w}}\frac{e^{-\beta\overline{w}}}{\overline{q}(s;\overline{w})}\mathrm{d}s.$$

Because $\lim_{\overline{w}\uparrow\overline{w}_{\max}} e^{-\beta\overline{w}} = e^{-\beta\overline{w}_{\max}}$ is finite, it suffices to show

$$\lim_{\overline{w}\uparrow\overline{w}_{\max}}\int_0^{\overline{w}}\frac{1}{\overline{q}(s;\overline{w})}\mathrm{d}s = \infty.$$

Substitute in the definition of $\overline{q}(s;\overline{w})$ in (16), this integral has a closed-form solution and

$$\lim_{\overline{w}\uparrow\overline{w}_{\max}}\int_0^{\overline{w}}\frac{1}{\overline{q}(s;\overline{w})}\mathrm{d}s = \lim_{\overline{w}\uparrow\overline{w}_{\max}}\frac{1}{r\kappa}\ln\left(\frac{\rho(r+\lambda)}{\rho(r+\lambda)-r\kappa\left(e^{(r+\lambda)\overline{w}}-1\right)\left(\rho+e^{-\lambda\overline{w}}\left(1-\rho\right)\right)}\right).$$

Recall that $\overline{w}_{\max}$ is given by $\overline{q}(0;\overline{w}_{\max}) = 0$ (and $\overline{q}(0;\overline{w}_{\max}) > 0$ for $\overline{w} < \overline{w}_{\max}$). It follows directly from rearranging the equation $\overline{q}(0;\overline{w}_{\max}) = 0$ that as $\overline{w}$ increasing to $\overline{w}_{\max}$, the denominator above goes to zero from above. That is,

$$\lim_{\overline{w}\uparrow\overline{w}_{\max}} \rho(r+\lambda) - r\kappa\left(e^{(r+\lambda)\overline{w}} - 1\right)\left((1-\rho)\,e^{-\lambda\overline{w}} + \rho\right) = 0^{+}.$$

The result follows. ☐

It follows from the claim that $\lim_{\overline{w}\uparrow\overline{w}_{\max}} \overline{G}_I(0;\overline{w}) = -\infty$. By the intermediate value theorem, there exists a unique $w^* \in (0, \overline{w}_{\max})$ such that $\overline{G}_I(0; w^*) = 0$. ☐

Lemma 17. *Denote $G_I^*(w) = \overline{G}_I(w; w^*)$. Then pair w^* and $G_I^*(w)$ uniquely solves (BVP).*

Proof. (BVP) subsumes (BVP-1) with the additional boundary condition $G_I(0) = 0$. By Lemma 16, $G_I^*(w)$ is the solution to (BVP-1) such that $G_I^*(0) = 0$. The result follows. ☐

To sum up, by Lemma 16, w^* is unique. By Lemma 15 and Lemma 17, the pair w^* and $G_I^*(w)$ solves (BVP). The theorem follows.

A.1.3 (IV) Sufficient conditions

To establish sufficiency, one needs to show that the (unique) solution to the boundary value problem (BVP) is a proper probability distribution and satisfies the equilibrium conditions. Because the solution must satisfy the boundary conditions, $G_I(0) = 0$ and $G_I(\overline{w}) = 1$, to show the solution is a proper probability distribution function, it remains to show the solution is strictly increasing. This is formalized in the following lemma.

Lemma 18. *Let the pair w^* and $G_I^*(w)$ be the solution to the boundary value problem (BVP). Then $G_I^*(w)$ is strictly increasing in w for $w \in [0, w^*]$.*

Proof. By Lemma 15, I consider the solution to (BVP-q). Take the derivative of (17) with respect to w on both sides,

$$\beta G_I^{*\prime}(w) = G_I^{*\prime\prime}(w) + \frac{\beta\rho}{\rho-\eta}\eta e^{\lambda w}\left(\lambda\left(1 - \frac{1}{q^*(w)}\right) + \frac{q^{*\prime}(w)}{q^*(w)^2}\right).$$

Because at $w = w^*$, $G_I^{*\prime}(w^*) = 0$ and $q^*(w^*) = p(w^*)$, moreover, $q^{*\prime}(w) = r\kappa + (r+\lambda)q^*(w)$, so

$$G_I^{*\prime\prime}(w^*) = -\frac{\beta\rho}{\rho-\eta}\eta e^{\lambda w^*}\left(\frac{\lambda p(w^*)^2 + r\kappa + rp(w^*)}{p(w^*)^2}\right) < 0,$$

where the inequality follows from $\rho > \eta$. Suppose there exists $\hat{w} < w^*$ such that $G_I^{*\prime}(\hat{w}) = 0$. Then

$$G_I^{*\prime\prime}(\hat{w}) = -\frac{\beta\rho}{\rho-\eta}\eta e^{\lambda\hat{w}}\left(\lambda\left(1 - \frac{1}{q^*(\hat{w})}\right) + \frac{r\kappa + (r+\lambda)q^*(\hat{w})}{q^*(\hat{w})^2}\right). \tag{21}$$

As is shown in the proof of Lemma 15, $q(\hat{w}) > 0$, so $r\kappa + (r+\lambda)q(\hat{w}) > \lambda\,(1 - q^*(\hat{w}))\,q^*(\hat{w})$. This implies the term in the parenthesis of (21) is positive. By $\rho > \eta$, $G_I^{*\prime\prime}(\hat{w}) < 0$.

This says that if there exists $\hat{w} < w^*$ such that $G_I^{*\prime}(\hat{w}) = 0$, then it must be $G_I^{*\prime\prime}(\hat{w}) < 0$. This is a contradiction because as shown, $G_I^{*\prime}(w)$ decreases to 0 at w^*, so if there exists $w < w^*$ at which $G_I^{*\prime}(w) = 0$, it must be either increasing to zero from below or tangent to zero. □

A.1.4 Proof of Lemma 2

The proof is subsumed by the proof of Lemma 12 and Lemma 13 above.

A.1.5 Proof of Lemma 3

I first prove the result for η. I first show that for a fixed $\overline{w}$, the solution to (BVP-2) is increasing in η. Let $\overline{G}_I(w;\overline{w};\eta)$ denote the solution to (BVP-2) given $\overline{w}$ and η. For any two η_1 and η_2 where $\eta_1 < \eta_2$, the boundary condition in (BVP-2) says $\overline{G}_I(w;\overline{w};\eta_1) = \overline{G}_I(w;\overline{w};\eta_2) = 1$. Now I show the differential equation (17) in (BVP-2),

$$G_I'(w;\overline{w};\eta) = \beta G_I(w;\overline{w};\eta) - \frac{\beta\rho}{\rho-\eta}\left(1 - \eta + e^{\lambda w}\eta\left(1 - \frac{1}{\overline{q}(w;\overline{w})}\right)\right)$$

is increasing in η for a fixed $G_I(w;\overline{w};\eta)$ for all $w < \overline{w}$. The derivative of the right-hand side of the differential equation with respect to η is equal to

$$-\frac{\beta\rho}{(\rho-\eta)^2}\left(1 - \rho\left(1 + e^{\lambda w}\left(\frac{1}{\overline{q}(w;\overline{w})} - 1\right)\right)\right) > 0,$$

where the inequality follows from plugging in the definition of $\overline{q}(w;\overline{w})$ given by (16). By a standard comparison argument (see Teschl, 2012, Theorem 1.3), the solution to (BVP-2), $\overline{G}_I(w;\overline{w};\eta)$, is decreasing pointwise in η for $[0,\overline{w}]$.

Let $w^*(\eta_1)$ be the (unique) solution such that $\overline{G}_I(0;w^*(\eta_1);\eta_1) = 0$. The above argument implies $\overline{G}_I(0;w^*(\eta_1);\eta_2) < 0$. Let $w^*(\eta_2)$ be the (unique) solution such that $\overline{G}_I(0;w^*(\eta_2);\eta_2) = 0$. Recall that by (20), for a fixed η, $\partial\overline{G}_I(0;\overline{w};\eta)/\partial\overline{w} < 0$. So it must be that $w^*(\eta_2) < w^*(\eta_1)$.

The proof for ρ is analogous. Only that one needs to show the differential equation (17) in (BVP-2) is decreasing in ρ. Below is the proof.

By definition (18), differential equation (17) can be written as $G_I'(w;\overline{w};\rho) = \beta G_I(w;\overline{w};\rho) - H(w;\overline{w};\rho)$. Showing this differential equation is decreasing in ρ for a fixed $G_I(w;\overline{w};\rho)$ is equivalent to showing $H(w;\overline{w};\rho)$ is increasing in ρ for a fixed $G_I(w;\overline{w};\rho)$. Take the derivative of H with respect to w. Because $\overline{q}(w;\overline{w})$ is a solution to (BVP-q), so $\partial\overline{q}(w;\overline{w};\rho)/\partial w =$

$r\kappa + (r+\lambda)\overline{q}(w;\overline{w};\rho)$, then

$$\frac{\partial}{\partial w}H(w;\overline{w};\rho) = e^{\lambda w}\frac{\beta\rho\eta}{\rho-\eta}\left(\lambda + \frac{r\kappa}{\overline{q}(w;\overline{w};\rho)^2} + \frac{r}{\overline{q}(w;\overline{w};\rho)}\right).$$

Because $\overline{q}(w;\overline{w};\rho)$ is increasing in ρ and $\beta\rho\eta/\left(\rho-\eta\right)$ is decreasing in ρ, so $\partial H(w;\overline{w};\rho)/\partial w < 0$. Because $H(\overline{w};\overline{w};\rho) = \beta$ for all ρ, by a standard comparison argument (see Teschl, 2012, Theorem 1.3), $H(w;\overline{w};\rho)$ is increasing pointwise in ρ. The result follows.

A.1.6 Proof of Proposition 1

Recall that $w^* \geq 0$ and $\rho \geq \eta$. Proving Proposition 1 is equivalent to showing $w^* = 0$ if and only if $\rho = \eta$. Fix $\rho = \eta$. The uninformed agent's expected payoff from waiting $w \geq 0$ is

$$\begin{aligned}
U(w) =& e^{-\beta w}\left(\rho + e^{-\lambda w}(1-\rho)\right)e^{-rw}\left(\kappa + \frac{\rho}{\rho + e^{-\lambda w}(1-\rho)}\right)\\
&+ e^{-\beta w}(1-\rho)\int_0^w \lambda e^{-\lambda s}e^{-rs}\kappa \mathrm{d}s\\
&+ \int_0^w \beta e^{-\beta\tau}\left(\rho + e^{-\lambda\tau}(1-\rho)\right)e^{-r\tau}\left(\kappa + \frac{\rho}{\rho + e^{-\lambda\tau}(1-\rho)}\right)\mathrm{d}\tau\\
&+ \int_0^w \beta e^{-\beta\tau}\left((1-\rho)\int_0^\tau \lambda e^{-\lambda s}e^{-rs}\kappa \mathrm{d}s\right)\mathrm{d}\tau.
\end{aligned}$$

It can be readily verified that $U(w)$ is strictly decreasing in w for all $w \geq 0$ and therefore maximized at $w = 0$. The same calculation and conclusion apply to $V(w)$.

A.1.7 Preliminaries for Results on Delayed Disclosure Equilibrium

With $\kappa = 0$, I derive an equation that characterizes w^* for any start time $t > 0$. This characterization reduces many of the proofs in the starting game to algebraic simplifications.

The boundary value problem in the continuation stopping game becomes

$$G_I''(w) = (\beta - r)G_I'(w) + \beta r G_I(w) - \frac{\beta\rho}{\rho-\eta}\left(e^{\lambda w}\eta(\lambda + r) + r(1-\eta)\right)$$

with boundary conditions $G_I(0) = 0$, $G_I(w^*) = 1$, and $G_I'(w^*) = 0$. This boundary value problem has a closed-form solution which yields an equation that pins down w^*. Let $\beta \neq \lambda$.[21]

Suppose disclosure starts at $t > 0$.[22] With a slight abuse of notation, denote the equilibrium waiting time in the continuation stopping game by $w^*(t)$.

[21] If $\beta = \lambda$, the expressions need to be written differently, but it does not affect the logic of the proof.

[22] Recall that if disclosure starts at $t = 0$, the agent will stop disclosing immediately, and the action the decision maker takes is μ. The ODE in the continuation game is defined only when $t > 0$.

Fix $\eta(t) = \mu$ for all t, also note that ρ is given by $\rho(t)$. Then w^* is given by

$$(\beta + r)(\beta - \lambda(1-\mu)) = \beta e^{-\lambda t + rw^*}(\beta - \lambda)\left(1 + \left(e^{\lambda(t+w^*))} - 1\right)\mu\right) + e^{-\beta w^*}\left(e^{-\lambda t} r(\beta - \lambda)(1-\mu) + \beta e^{\lambda w^*}(r+\lambda)\mu\right). \tag{22}$$

This is a special case of the equilibrium analyzed in the main model. It can be verified that for each t, there exist a unique $w^* > 0$ that satisfies this equation. Note that the right-hand side is continuous and differentiable function in w^* and in t. Even though this equation does not have a closed-form solution, it implicitly pins down w^* as a continuous and differentiable function of t. Denote the solution to this equation by $w^*(t)$. In words, this is the equilibrium waiting time in the continuation stopping game if disclosure starts at t.

In the stopping game, the agent's equilibrium payoff conditional on $\theta = 0$ and $\theta = 1$ are

$$V^*(t) = e^{-\lambda w^*(t)} e^{-rw^*(t)} \frac{\rho(t)}{\rho(t) + (1-\rho(t))e^{-\lambda w^*(t)}} \text{ and } U_1^*(t) = V^*(t)\frac{\beta - \lambda e^{(\lambda-\beta)w^*(t)}}{\beta - \lambda}.$$

An uninformed agent's equilibrium payoff is $U^*(t) = (1-\rho(t))V^*(t) + \rho(t)U_1^*(t)$. By (22), $w^*(t)$ is differentiable in t, so $V^*(t)$ and $U_1^*(t)$ are also differentiable, which then implies $Y(t)$ is also differentiable in t, where

$$Y(t) = \left(\int_0^t \lambda e^{-\lambda s} e^{-rs} V^*(s) \mathrm{d}s + e^{-rt} e^{-\lambda t} V^*(t)\right) + e^{-rt} U_1^*(t).$$

Denote the informed agent's expected payoff from delaying till t by $Z(t)$, then

$$Z(t) = e^{-rt} V^*(t).$$

The proofs of the following two results are mostly algebraic and are in the Online Appendix.

Lemma 19. *Given $\eta(t) = \mu$ for all $t \geq 0$, $V^*(t)$ is decreasing in t for all $t \geq 0$.*

Lemma 20. *If $Y(t)$ is increasing (decreasing), then $Y(t|t_0)$ is increasing (decreasing). Similarly, if $Z(t)$ is increasing (decreasing), then $Z(t|t_0)$ is increasing (decreasing).*

Immediate disclosure is an equilibrium if and only if $Y(t)$ is decreasing in t for all $t \geq 0$ and $Z(t)$ is decreasing in t for all $t \geq 0$. By these two lemmas, the informed agent starts disclosing as soon as he gets the opportunity to do so. I now focus on the uninformed agent.

A.1.8 Proof of Proposition 2

I derive conditions under which immediate disclosure is an equilibrium if disclosure stops as soon as it starts and the decision maker takes the highest possible action $\rho(t)$. I then show under these conditions, immediate disclosure is an equilibrium in the main model.

Let $Y_0(t)$ denote the uninformed agent's expected payoff from waiting till t if remains uninformed and starting immediately if becomes informed in the best-case scenario, then

$$Y_0(t) = (1-\mu)\int_0^t \lambda e^{-\lambda s} e^{-rs}\rho(s)\mathrm{d}s + \left((1-\mu)e^{-\lambda t} + \mu\right) e^{-rt}\rho(t).$$

The derivative is proportional to $\rho'(t) - r\rho(t)$. So $Y_0(t)$ is decreasing for all $t \geq 0$ if and only if $\rho'(t) - r\rho(t) \leq 0$ for all $t \geq 0$. Note that this function is decreasing in t for all t, so $\rho'(t) - r\rho(t) \leq 0$ for t if and only if $\rho'(0) - r\rho(0) \leq 0$, which is equivalent to $r/\lambda \geq 1-\mu$. The proof for the informed agent is analogous. The proposition follows from the next lemma. The proof of the lemma is mostly algebraic and is relegated to the Online Appendix.

Lemma 21. *If $\rho'(t) - r\rho(t) \leq 0$ for all $t \geq 0$, then $Y'(t) \leq 0$ for all $t \geq 0$.*

A.1.9 Proof of Lemma 4

Take the derivative of $Y(t)$ with respect to t. The derivative is proportional to

$$Y'(t) \propto y(t) := (1-\mu)e^{-\lambda t}\left(V^{*\prime}(t) - rV^*(t)\right) + \mu\left(U_1^{*\prime}(t) - rU_1^*(t)\right).$$

Differentiate both sides of equation (22) with respect to t, with respect to t and rearrange,

$$w^{*\prime}(t) = \frac{\beta e^{(\beta+r)w^*(t)} + r}{\beta\left(e^{(\beta+r)w^*(t)} - 1\right)} \frac{\lambda(1-\mu)}{r(1-\mu) + e^{\lambda(t+w^*(t))}(r+\lambda)\mu}. \tag{23}$$

By (23), $w^*(t)$ and $w^{*\prime}(t)$ are bounded for all $t > 0$, so $V^{*\prime}(t), V^*(t), U_1^{*\prime}(t)$, and $U_1^*(t)$ are also bounded. Plug (23) into $y(t)$, then take the limit of $y(t)$ as $r \to 0$. After simplifying,

$$\begin{aligned}
&\lim_{r\to 0}\left((1-\mu)e^{-\lambda t}\left(V^{*\prime}(t) - rV^*(t)\right) + \mu\left(U_1^{*\prime}(t) - rU_1^*(t)\right)\right) \\
&\quad = \frac{\lambda(1-\mu)\mu^2 e^{\lambda t - \beta w^*(t)}}{\left(e^{\beta w^*(t)} - 1\right)\left(\mu\left(e^{\lambda(w^*(t)+t)} - 1\right) + 1\right)^2}\left(e^{(\beta+\lambda)w^*(t)} + \frac{\lambda e^{\lambda w^*(t)} - \beta e^{\beta w^*(t)}}{\beta - \lambda}\right).
\end{aligned}$$

It can be readily verified that this expression is positive for all $w^*(t) > 0$.

A.1.10 Proof of Lemma 5

By definition, $\lim_{t\to\infty}\rho'(t) = 0$. By (23), $w^*(t) < \infty$ for all t and $\lim_{t\to\infty} w^{*\prime}(t) = 0$. Moreover, $\partial V^*/\partial\rho$, $\partial V^*/\partial w^*$, $\partial U_1^*/\partial\rho$, and $\partial U_1^*/\partial w^*$ are all finite for all t. As a result,

$$\lim_{t\to\infty} V^{*\prime}(t) = \frac{\partial V^*}{\partial\rho}\rho'(t) + \frac{\partial V^*}{\partial w^*}w^{*\prime}(t) = 0 \text{ and } \lim_{t\to\infty} U_1^{*\prime}(t) = \frac{\partial U_1^*}{\partial\rho}\rho'(t) + \frac{\partial U_1^*}{\partial w^*}w^{*\prime}(t) = 0.$$

Because $V^*(t) > 0$ and $U_1^*(t) > 0$ for all t, $\lim_{t\to\infty} y(t) < 0$.

A.1.11 Proof of Theorem 1.A

Define $\tau^* := \inf\{\tilde{t} \geq 0 : Y'(t) \leq 0 \text{ for all } t \geq \tilde{t}\}$. By Lemma 5, $\tau^* < \infty$. I show that neither type of agent starts at $t < \tilde{\tau}$, and both types starts immediately at $t \geq \tilde{\tau}$. Suppose disclosure starts at $\tilde{\tau} \geq \tau^*$. If the agent deviates to starting at $t < \tilde{\tau}$, the decision maker's belief is $\theta = 0$. The agent stops immediately, the decision maker takes action 0, so the agent's payoff is 0. If the agent starts at $t \geq \tilde{\tau}$, his equilibrium payoff in the continuation game is $V^*(t)$ if he is informed and $U^*(t)$ if he is uninformed. Both are strictly positive for all t as the decision maker's action if disclosure stops at any w is $q(w) > 0$. So the agent does not deviate to starting at $t < \tilde{\tau}$. At any $t \geq \tilde{\tau}$, by definition of τ^*, $Y(t)$ is decreasing in t and $Z(t)$ is decreasing in t as shown before, so both types of agent start disclosing immediately.

A.2 Proofs for Section 4

A.2.1 Proof of Proposition 3

Because both types of the agent adopt the same starting strategy, the decision maker's belief that $\theta = 1$ if disclosure starts at any time t (that is on the equilibrium path) is μ. By Lemma 3, w^* is increasing in ρ, which is increasing in τ, which means w^* is increasing in τ.

As the disclosure time τ increases, the uninformed agent's belief at the beginning of the stopping game approaches 1 and the decision maker's belief is $\eta = \mu$. One can obtain the upper bound $\overline{w}$ by solving the boundary value problem (BVP) evaluated at $\rho = 1$.

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